SESSION 5

Analytical Geometry

Nov 2019

Q3.1

Equation of PR: y = 5

Q3.2.1

$m_{RS} = \frac{5-(-7)}{3-(-3)} = \frac{12}{6}$

$= 2$

Q3.2.2

$m_{RS} = m_{PT}$ [PT ∥ RS]

$\tan\theta = 2$

$\theta = 63.43°$

Q3.2.3

$m_{RS} = m_{RD} = m_{DS}$

$2 = \frac{5-y}{3-0} = \frac{y+7}{0-(-3)}$

$\therefore y = -1$

$\therefore D(0 ; -1)$

Q3.3

$ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$

$20 = 4 + (k+7)^2$

$(k+7)^2 = 16$

$k + 7 = \pm 4$

$k = -11$ or $k = -3$

$\therefore k = -3$

Q3.4

Method: translation

T→S:

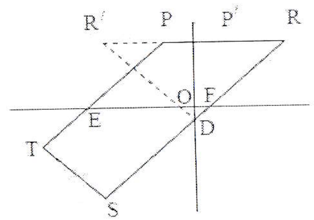
$(x ; y) \to (x+2 ; y-4)$

∴ by symmetry: D→N:

$D(0 ; -1) \to N(0+2 ; -1-4)$

$\therefore N(2 ; -5)$

Q3.5



β is the inclination of RS $\therefore \beta = 63.434...°$

$O\hat{F}D = 63.434...°$ [vert opp ∠s]

$O\hat{D}F = 90° - 63.434...° = 26.565...°$

$R\hat{D}R' = 2(26.565...°) = 53.13°$

Q4.1

$M(-1 ; 1)$

$(x+1)^2 + (y-1)^2 = 1$

Q4.2

Midpoint of CB, N: (−0,5 ; 1,5)

$\therefore \frac{x_C + 0}{2} = -\frac{1}{2}$ and $\frac{y_C + 1}{2} = \frac{3}{2}$

$\therefore C(-1 ; 2)$

Q4.3

$m_{radius} = \frac{2-1}{-1-0}$

$= -1$

$\therefore m_{tangent} = 1$

$y - y_1 = m(x - x_1)$

$y - y_1 = 1(x - x_1)$

$y - 2 = 1(x - (-1))$

$y - 2 = x + 1$

$\therefore y = x + 3$

$y - x = 3$

Q4.4

Tangents to circle:

$y = x + 3$ and $y = x + 1$

$\therefore t > 3$ or $t < 1$

Q4.5

$D(-3 ; 0)$

C→N:

$(x ; y) \to (x+0,5 ; y-0,5)$

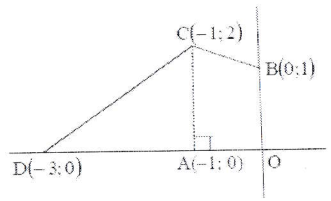
D→E:

$D(x ; y) \to E(x+0,5 ; y-0,5)$

$\therefore E(-3+0,5 ; 0-0,5)$

$\therefore E(-2,5 ; -0,5)$

Q4.6



area of trapezium AOBC $= \frac{1}{2}(1+2)(1)$

$= 1\frac{1}{2}$ square units

area of ΔACD $= \frac{1}{2}(2)(2)$

$= 2$ square units

area of quadrilateral OBCD $= 3\frac{1}{2}$ square units

$\therefore 2a^2 = \frac{7}{2}$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

May-June 2019

Q3.1.1

Midpoint of EC:

$= \left(\frac{-2+2}{2} ; \frac{0+(-3)}{2}\right) = \left(0 ; \frac{-3}{2}\right)$

Q3.1.2

$m_{DC} = \frac{-3-(-5)}{2-(-2)}$

$= \frac{2}{4} = \frac{1}{2}$

Q3.1.3

$m_{AB} = \frac{1}{2}$

$y = \frac{1}{2}x + c$

$0 = \frac{1}{2}(-2) + c$

$c = 1$

$\therefore y = \frac{1}{2}x + 1$

Q3.1.4

$\tan\alpha = m_{AB} = \frac{1}{2}$

$\alpha = 26{,}57°$

$\theta = 90° + 26{,}57°$

$= 116{,}57°$

Q3.2

B(0 ; 1)

$m_{BC} = \frac{1-(-3)}{0-2}$ $\quad m_{AB} \times m_{BC} = \frac{1}{2} \times -2$

$= -2$ $\quad = -1$

$\therefore AB \perp BC$

Q3.3.1

$A\hat{B}C = 90°$

$\therefore$ EC is diameter [converse: $\angle$ in semi circle]

$\therefore$ centre of circle $= \left(0\ ;\ -\frac{3}{2}\right)$

Q3.3.2

$(x-0)^2 + \left(y+\frac{3}{2}\right)^2 = r^2$

$(-2-0)^2 + \left(0+\frac{3}{2}\right)^2 = r^2$

$\therefore\ r^2 = \frac{25}{4}$ or $r = \frac{5}{2}$

$x^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{4}$

Q4.1

$(x-2)^2 + (y-1)^2 = 25$

$(-2-2)^2 + (b-1)^2 = 25$

$(b-1)^2 = 9$

$b - 1 = \pm 3$

$\therefore b = 4$ or $b \neq -2$

Q4.2.1

K(2 ; 1 − 5)

$\therefore$ K(2 ; −4)

Q4.2.2

$m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$

$m_{PL} = \frac{4}{3}$ [radius $\perp$ tangent]

$y - y_1 = \frac{4}{3}(x - x_1)$

$y - 4 = \frac{4}{3}(x + 2)$

$y = \frac{4}{3}x + \frac{20}{3}$

Q4.2.3

$y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$

$L\left(2\ ;\ \frac{28}{3}\right)$ and K(2 ; −4): $LK = \frac{28}{3} - (-4) = \frac{40}{3}$

Coordinates of P:

$\frac{x+2}{2} = -4\frac{1}{2}$ and $\frac{y-4}{2} = -6$

$\therefore x = -11$ $\quad y = -8$

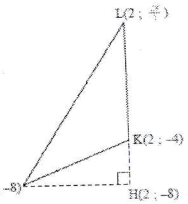
$\therefore$ P(−11; −8)

$\perp$height (PH) = 2 − (−11) = 13

Area ΔPKL $= \frac{1}{2}$(LK)(PH)

$= \frac{1}{2}\left(\frac{40}{3}\right)(13)$

$= \frac{260}{3}$ OR 86,67 square units



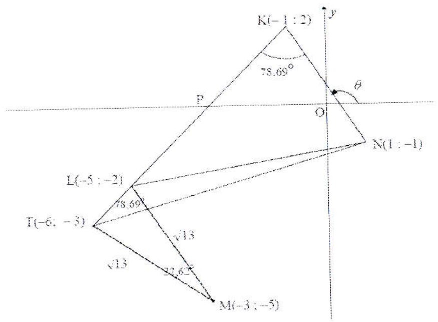
Q4.3

The centres of the two circles lie on the same vertical line $x = 2$, and the sum of the radii = 10

$n - 1 = 10$ or $1 - n = 10$

$n = 11$ $\quad n = -9$

__November 2018__



Q3.1.1

$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{KN} = \frac{2-(-1)}{-1-1}$

$= -\frac{3}{2}$

Q3.1.2

$\tan\theta = m_{KN} = -\frac{3}{2}$

$\theta = 180° - 56{,}31°$

$\theta = 123{,}69°$

Q3.2

Inclination KL = 123,69° − 78,69° = 45°

$\tan 45° = m_{KL} = 1$

Q3.3

$y - y_1 = 1(x - x_1)$

$y - 2 = 1(x - (-1))$

$y = x + 3$

Q3.4

$KN = \sqrt{(1+1)^2 + (-1-2)^2}$

$KN = \sqrt{13}$ or 3,61

Q3.5.1

$(x+3)^2 + (y+5)^2 = 13$ ...(1)

L is a point on KL

$y = x + 3$ ...(2)

(2) in (1):

$(x+3)^2 + (x+3+5)^2 = 13$

$x^2 + 6x + 9 + x^2 + 16x + 64 = 13$

$2x^2 + 22x + 60 = 0$

$x^2 + 11x + 30 = 0$

$(x+5)(x+6) = 0$

$x = -5$ or $x = -6$

$y = -2$ or $y = -3$

L(−5 ; −2) or (−6 ; −3)

Q3.5.2

Midpoint of KM: (− 2 ; − 1,5)

$\therefore \frac{x_L + 1}{2} = -2$ and $\frac{y_L - 1}{2} = -\frac{3}{2}$

$\therefore$ L(− 5 ; − 2)

Q3.6

T(−6 ; −3) (from Question 3.5.1)

$KT = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$

$= \sqrt{50}$

$KN = \sqrt{13}$ (CA from 3.4)

Area of $\Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$

$= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$

= 12,50 square units

Q4.1

F(3 ; 1)

Q4.2

$FS = \sqrt{(6-3)^2 + (5-1)^2}$

FS = 5

Q4.3

FH(FS) : HG = 1 : 2

$\therefore$ HG = 2 FH

= 10

Q4.4

Tangents from common/same point

Q4.5.1

$F\hat{H}J = 90°$

$FJ^2 = 20^2 + 5^2$

$FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62

[tan ⊥ radius / *rkl ⊥ radius*]

[Pyth theorem/*stelling*]

Q4.5.2

$(x-m)^2 + (y-n)^2 = 100$

Q4.5.3

$K(22; n)$

GK = HG = 10

FH = FS = 5

$m = 22 - 10$

$m = 12$

Let $J(22 ; y)$:

$FJ^2 = (22-3)^2 + (y-1)^2$

$425 = 361 + y^2 - 2y + 1$

$0 = y^2 - 2y - 63$

$0 = (y-9)(y+7)$

$\therefore y = 9$ or/of $y \neq -7$

$\therefore n = 9 - 20 = -11$

$\therefore G(12; -11)$

<u>**June 2018**</u>

Q3.1

$$m_{AC} = \frac{1-(-4)}{7-(-3)} = \frac{5}{10} = \frac{1}{2}$$

Q3.2.1

$$y - y_1 = \frac{1}{2}(x - x_1)$$

$$y - 1 = \frac{1}{2}(x-7)$$

$$y - 1 = \frac{1}{2}x - \frac{7}{2}$$

$$y = \frac{1}{2}x - 2\frac{1}{2}$$

Q3.2.2

$$M\left(\frac{-2+8}{2} ; \frac{9+(-11)}{2}\right)$$

$\therefore M(3 ; -1)$

Equation of AC: $y = \frac{1}{2}x - 2\frac{1}{2}$

$y = \frac{1}{2}(3) - 2\frac{1}{2}$

$y = -1$

$\therefore$ M lies on AC

Q3.3

$$m_{BD} = \frac{9-(-11)}{-2-8} = -2$$

$$m_{BD} \times m_{AC} = \frac{1}{2} \times -2 = -1$$

$\therefore BD \perp AC$

Q3.4.1

$\tan\theta = m_{BD} = -2$

$\therefore \theta = 116{,}57°$

Q3.4.2

$\tan\beta = m_{BC}$

$$m_{BC} = \frac{9-(-4)}{-2-(-3)} = 13$$

$\beta = 85{,}6°$

$\therefore C\hat{B}D = 116{,}57° - 85{,}60°$ [ext ∠ of Δ]

$= 30{,}97°$

Q3.4.3

$$AC = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$= \sqrt{(7-(-3))^2 + (1-(-4))^2}$$

$$= \sqrt{100+25}$$

$$= \sqrt{125} = 5\sqrt{5} = 11{,}58$$

Q3.4.4

$$BM = \sqrt{((-2)-3)^2 + (9-(-1))^2}$$

$$= \sqrt{125} = 5\sqrt{5}$$

Area of ΔABC $= \frac{1}{2}$ base × ⊥ height

$= \frac{1}{2}(\sqrt{125})(\sqrt{125})$

= 62,5 square units

Area of ABCD = 2 × 62,5

= 125 square units

Q4.1

$$M\left(\frac{0+4}{2} ; \frac{0+(-6)}{2}\right)$$

$\therefore M(2 ; -3)$

Q4.2.1

$x^2 + y^2 = 4^2 + (-6)^2$

$= 52$

$\therefore x^2 + y^2 = 52$

Q4.2.2

$$(x-2)^2 + (y+3)^2 = \left(\frac{\sqrt{52}}{2}\right)^2 = 13$$

$x^2 - 4x + 4 + y^2 + 6y + 9 - 13 = 0$

$x^2 + y^2 - 4x + 6y = 0$

Q4.2.3

$$m_{OP} = \frac{-6}{4} = -\frac{3}{2}$$

$m_{RS} \times m_{OP} = -1$

$\therefore m_{RS} = \frac{2}{3}$

$\therefore y = \frac{2}{3}x$

Q4.3

$x^2 + y^2 = 52 \quad and \quad y = \frac{2}{3}x$

$x^2 + \left(\frac{2}{3}x\right)^2 = 52$

$x^2 + \frac{4}{9}x^2 = 52$

$1\frac{4}{9}x^2 = 52$

$x^2 = 36$

$x = 6$

$\therefore$ R(6 ; 4) and N(−6 ; 4)

$\therefore$ NR = 12 units

Q4.4

Let T(x ; 0) be the other x intercept of the small circle
Then OT is the common chord

$\therefore (x-2)^2 + (0+3)^2 = 13$

$(x-2)^2 = 13 - 9 = 4$

$x - 2 = \pm 2$

$x = 2 \pm 2$

$x = 4 \quad or \quad 0$

$\therefore$ length of common chord = OT = 4 units

March 2018

Q3.1

x=3

Q3.2

$m_{QP} = \tan 71{,}57°$

$= 3$

Q3.3

$y - y_1 = m(x - x_1)$

$y + 2 = 3(x + 7)$

$y = 3x + 19$

Q3.4

R(3 ; 0)

$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-7-3)^2 + (-2-0)^2}$

$= \sqrt{104}$ or $2\sqrt{26}$

Q3.5

$\tan(90° - \theta) = m_{QR}$

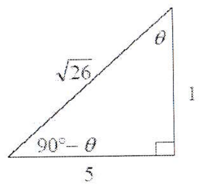
$= \frac{0-(-2)}{3-(-7)}$

$= \frac{1}{5}$

Q3.6

$RN = \frac{1}{2}.2\sqrt{26} = \sqrt{26}$

SR = 6



Area $\Delta RSN = \frac{1}{2} SR \,.\, RN.\ \sin\theta$

$= \frac{1}{2} \times 6 \times \sqrt{26} \times \frac{5}{\sqrt{26}}$

= 15 square units

Q4.1

$OK = \sqrt{180}$ or $6\sqrt{5}$

Q4.2

$a^2 + b^2 = 180$

$a = 2b$

$(2b)^2 + b^2 = 180$

$5b^2 = 180$

$b^2 = 36 \qquad \therefore b = -6$

$a = 2(-6)$

K (−12 ; − 6) (given)

Q4.3.1

$m_{OK} = \frac{1}{2}$

$m_{PT} = -2$

$y = mx + c$

$-6 = -2(-12) + c$

$c = -30$

$y = -2x - 30$

Q4.3.2

3MK = OK

$\Rightarrow OM = \frac{4}{3}OK$

$M = \frac{4}{3}(-12 ; -6)$

$\therefore M(-16 ; -8)$

Q4.3.3

$(x - (-16))^2 + (y - (-8))^2 = \left(\frac{1}{3}\sqrt{180}\right)^2$

$(x+16)^2 + (y+8)^2 = 20$

Q4. 4

$OK < r < OK + 2KM$

$\sqrt{180} < r < \sqrt{180} + \frac{2}{3}\sqrt{180}$

$6\sqrt{5} < r < 10\sqrt{5}$

Q4.5

$x^2 + 32x + (16)^2 + y^2 + 16y + (8)^2 = 256 + 64 - 240$

$(x+16)^2 + (y+8)^2 = 80$

New circle/*nuwe sirkel*:
Centre/*middelpt* (−16 ; − 8) &

$r = 4\sqrt{5}$

Original circle/*oorspronklike sirkel*:

M(−16 ; − 8) & $r = 2\sqrt{5}$

This circle will never cut the circle with centre M as they have the **same centre (concentric circles) but unequal radii**/*Hierdie sirkel sal nooit die sirkel met middelpt M sny nie, want hulle is* ***konsentries****, want het* ***dieselfde middelpunt met verskillende radii***.

November 2017:

Q3.1.1

$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3\frac{1}{2} - (-4)}{3 - 8}$

$= -\frac{3}{2}$

$(y - (-4)) = -\frac{3}{2}(x - 8)$

$y + 4 = -\frac{3}{2}x + 12$

$y = -\frac{3}{2}x + 8$

Q3.1.2

AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$

$15x + 10y = 80$

$7x - 10y = 8$

$22x = 88$

$x = 4$

$3(4) + 2y = 16$

$y = 2$

$\therefore G(4 ; 2)$

Q3.2

$\frac{x_A + 4}{2} = 3$ and $\frac{y_A + 2}{2} = 3\frac{1}{2}$

$\therefore A(2 ; 5)$

Q3.3

The coordinates of the midpt of AB:

$\left(\frac{2+(-6)}{2} ; \frac{5+(-5)}{2}\right) = (-2 ; 0)$

But the y-coordinate of E is 0

$\therefore$ E(−2 ; 0) is the midpoint of AB

$\therefore$ EF || BG [midpoint theorem

Q3.4

Midpoint of AC $= \left(5 ; \frac{1}{2}\right)$

$\frac{x_D + (-6)}{2} = 5$ and $\frac{y_D + (-5)}{2} = \frac{1}{2}$

$\therefore D(16 ; 6)$

Q4.1.1

$m_{PK} = \frac{5-(-3)}{-4-0}$

$= -2$

PK ⊥ SR [radius ⊥ tangent

$\therefore m_{PK} \times m_{RS} = -1$

$\therefore m_{RS} = \frac{1}{2}$

Q4.1.2

$(y-5) = \frac{1}{2}(x-(-4))$

$(y-5) = \frac{1}{2}x + 2$

$y = \frac{1}{2}x + 7$

Q4.1.3

$M\left(\frac{-4+0}{2} ; \frac{5+(-3)}{2}\right)$

$\therefore M(-2 ; 1)$

$r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$r^2 = (-2+4)^2 + (1-5)^2$

$\therefore r^2 = 20$

$\therefore (x+2)^2 + (y-1)^2 = 20$ *or* $(\sqrt{20})^2$

Q4.1.4

$\tan\theta = m_{PK} = -2$

$\therefore \theta = 180° - 63.43°$

$= 116.57°$

$P\hat{K}R = 116.57° - 90°$ [ext ∠ of ΔMOK]

$= 26.57°$

Q4.1.5

RS || tangent at K(0 ; −3)

$\therefore m_{PS} = m_{tang} = \frac{1}{2}$

$\therefore y = \frac{1}{2}x - 3$

Q4.2

$t \in (-3 ; 7)$

Q4.3

RS: $y = \frac{1}{2}x + 7$ $\therefore S(-14 ; 0)$

$SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$

Area ΔSMK $= \frac{1}{2}$. MK . SP

$= \frac{1}{2}(\sqrt{20})(\sqrt{125})$

= 25 square units

June 2017:

Q3.1

$m_{CD} = \frac{-3-(-5)}{4-0}$

$= \frac{-3+5}{4-0}$

$= \frac{1}{2}$

Q3.2

$m_{AD} = \frac{-5-3}{0-(-4)}$

$= -2$

$m_{CD} \times m_{AD} = \frac{1}{2} \times -2$

$= -1$

$\therefore$ AD ⊥ DC

Q3.3

$AB = \sqrt{(3+4)^2 + (4-3)^2} = \sqrt{50} = 5\sqrt{2}$

$BC = \sqrt{(4-3)^2 + (-3-4)^2} = 5\sqrt{2}$

AB = BC

$\therefore$ ΔABC is an iscosceles triangle

Q3.4

[BF || DC]

$y - 4 = \frac{1}{2}(x-3)$

$y - 4 = \frac{1}{2}x - 1\frac{1}{2}$

$y = \frac{1}{2}x + 2\frac{1}{2}$

Q3.5

$\tan\alpha = -2$

$\therefore \alpha = 116.57°$

$\alpha = 90° + \theta$

$\therefore \theta = 26.57°$

Q3.6