ANSWERS TRIGONOMETRY SATURDAY CLASSES

Trigonometry

Nov 2019

Q5.1

$$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$$

$$=\frac{\sin x}{\cos x.\frac{\sin x}{\cos x}}+(-\sin x)\sin x$$

$$=1-\sin^2 x$$

$$=\cos^2 x$$

Q5.2

$$\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$$

$$=\frac{-\left(\cos^2 35°-\sin^2 35°\right)}{2(2\sin 10°\cos 10°)}$$

$$=\frac{-\cos 70°}{2\sin 20°}$$

$$=\frac{-\cos 70°}{2\cos 70°} \quad \text{OR} \quad =\frac{-\sin 20°}{2\sin 20°} \quad =-\frac{1}{2}$$

Q5.3

$$2\sin^2 77°=2[\sin(90°-13°)]^2$$

$$=2\cos^2 13°$$

$$=2\cos^2 13°-1+1$$

$$=\cos 26°+1$$

$$=m+1$$

Q5.4.1

$$\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°=\tan 165°$$

$$\sin(x+25°-15°)=-0{,}2679\ldots\ OR\ -2+\sqrt{3}$$

$$\sin(x+10°)=-0{,}2679\ldots OR\ -2+\sqrt{3}$$

$x+10°=195{,}54°+k.360°$

$x=185{,}54°+k.360°;\ k\in Z$ or

or $x+10°=344{,}46°+k.360°$

$x=334{,}46°+k.360°; k\in Z$

Q5.4.2

$$f(x)=\sin(x+10°)$$

For minimum value of $\sin x$: $x=270°$

For minimum value of $\sin(x+10°)$: $x=260°$

May-June 2019

Q5.1.1

$\sin 191°$

$= -\sin 11°$

Q5.1.2

$\cos 22°$

$= \cos(2 \times 11°)$

$= 1 - 2\sin^2 11°$

Q5.2

$\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$

$= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$

$= -\cos x + \sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x\left(\frac{1}{\sqrt{2}}\right)\right)$

$= -\cos x + \sin x + \cos x$

$= \sin x$

Q5.3

$\sin P + \sin Q = \sin P + \cos P$

$(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$

$\sin^2 P + 2\sin P\cos P + \cos^2 P = \frac{49}{25}$

$2\sin P\cos P = \frac{49}{25} - 1$

$\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$

$= \frac{24}{25}$

November 2018

Q5.1.1

$k^2 = (\sqrt{5})^2 - 1^2$

$= 4$

$k = -2$

Q5.1.2a

$\tan\theta = -\frac{1}{2}$

Q5.1.2b

$\cos(180° + \theta) = -\cos\theta$

$= \frac{2}{\sqrt{5}}$

Q5.1.2c

$\sin(\theta + 60°) = \frac{a+b}{\sqrt{20}}$

$\text{LHS} = \sin\theta\cos 60° + \cos\theta\sin 60°$

$= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1 - 2\sqrt{3}}{2\sqrt{5}}$

$= \frac{1 - 2\sqrt{3}}{\sqrt{20}}$

Q5.1.3

$\tan\theta = -\frac{1}{2}$

$\therefore \theta = 180° - 26{,}57°$

$\therefore \theta = 153{,}43°$

$\tan(2\theta - 40°) = \tan[(2 \times 153{,}43°) - 40°]$

$= \tan 266{,}87°$

$= 18{,}3$

Q5.2

$$\text{LHS} = \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} \qquad \text{RHS} = 2\tan 2x$$

$$= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{\cos^2 x + 2\sin x\cos x + \sin^2 x - \cos^2 x + 2\sin x\cos x - \sin^2 x}{\cos^2 x - \sin^2 x}$$

$$= \frac{2(2\sin x\cos x)}{\cos^2 x - \sin^2 x}$$

$$= \frac{2\sin 2x}{\cos 2x}$$

$$= 2\tan 2x$$

$$= \text{RHS}$$

Q5.3

$$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$$

$$= \cos^2 38^\circ + \cos^2 39^\circ + \cos^2 40^\circ + \ldots + \cos^2 51^\circ + \cos^2 52^\circ$$

$$= \sin^2 52^\circ + \sin^2 51^\circ + \sin^2 50^\circ + \ldots + \cos^2 51^\circ + \cos^2 52^\circ$$

$$= 7(1) + \cos^2 45^\circ$$

$$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{or} \quad = 7 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 7\frac{1}{2}$$

June 2018

Q5.1.1

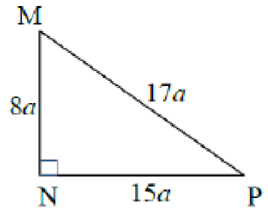
$$\text{Given}: \sin M = \frac{15}{17}$$

$$MN^2 = 17^2 - 15^2$$

$$= 64$$

$$MN = 8$$

OR



$$\therefore \tan M = \frac{15}{8}$$

Q5.1.2

$$\sin M = \frac{NP}{MP}$$

$$\frac{NP}{51} = \frac{15a}{17a}$$

$$\therefore NP = 45$$

Q5.2

$$\cos(x - 360^\circ).\sin(90^\circ + x) + \cos^2(-x) - 1$$

$$= \cos x \;.\; \cos x + \cos^2 x - 1$$

$$= \cos^2 x + \cos^2 x - 1$$

$$= 2\cos^2 x - 1$$

$$= \cos 2x$$

Q5.3.1

$$\sin(2x+40^\circ)\cos(x+30^\circ)-\cos(2x+40^\circ)\sin(x+30^\circ)$$
$$=\sin[(2x+40^\circ)-(x+30^\circ)]$$
$$=\sin(x+10^\circ)$$

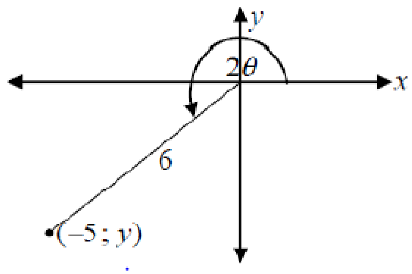
Q5.3.2

$$\sin(2x+40^\circ)\cos(x+30^\circ)-\cos(2x+40^\circ)\sin(x+30^\circ)=\cos(2x-20^\circ)$$
$$\therefore \cos(2x-20^\circ)=\sin(x+10^\circ)$$
$$\cos(2x-20^\circ)=\cos[90^\circ-(x+10^\circ)]$$
$$2x-20^\circ=80^\circ-x+k.360^\circ \text{ or } 2x-20^\circ=360^\circ-(80^\circ-x)+k.360^\circ$$
$$3x=100^\circ+k.360^\circ \text{ or } 2x-20^\circ=280^\circ+x+k.360^\circ$$
$$x=33{,}33^\circ+k.120^\circ \text{ or } x=300^\circ+k.360^\circ\ ;k\in Z$$

March 2018

Q5.1.1

$$\cos 2\theta=-\frac{5}{6},\text{ where } 2\theta\in[180^\circ;270^\circ]$$



$y^2=6^2-(-5)^2$ [Pythagoras]

$y=\pm\sqrt{11}$

$(5\ ;y)$ is in 3rd quadrant:

$\therefore y=-\sqrt{11}$

$$\sin 2\theta=-\frac{\sqrt{11}}{6}$$

Q5.1.2

$$\cos 2\theta=1-2\sin^2\theta$$
$$2\sin^2\theta=1-\cos 2\theta$$
$$\sin^2\theta=\frac{1-\left(-\frac{5}{6}\right)}{2}$$
$$=\frac{11}{6}\times\frac{1}{2}$$
$$=\frac{11}{12}$$

Q5.2

$\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$

=sinx.cosx + sinxcosx

=2sinxcosx

=sin2x

Q5.3

$\sin 3x.\cos y + \cos 3x.\sin y$

$\sin(3x + y)$

$= \sin 270°$

$= -1$

Q5.4.1

$2\cos x = 3\tan x$

$2\cos x = \frac{3\sin x}{\cos x}$

$2\cos^2 x = 3\sin x$

$2(1 - \sin^2 x) = 3\sin x$

$2 - 2\sin^2 x = 3\sin x$

$2\sin^2 x + 3\sin x - 2 = 0$

Q5.4.2

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -2$ (no solution)

$x = 30° + k.360°$ or $x = 150° + k.360°$; $k \in Z$

Q5.4.3

$5y = 30° + k.360°$ or $5y = 150° + k.360°$

$y = 6° + k.72°$ or $y = 30° + k.72°$

$\therefore y = 144° + 6°$ or $y = 144° + 30°$

$y = 150°$ or $y = 174°$

November 2017:

Q5.1

$$\frac{\sin(A-360^\circ).\cos(90^\circ+A)}{\cos(90^\circ-A).\tan(-A)}$$

$$=\frac{\sin A(-\sin A)}{\sin A(-\tan A)}$$

$$=\frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$$

$$=\cos A$$

Q5.2.1

$$t^2=(\sqrt{34})^2-(3)^2$$

$$\therefore t=-5$$

Q5.2.2

$$\tan\beta=\frac{-5}{3}$$

Q5.2.3

$$\cos 2\beta = 2\cos^2\beta-1$$

$$=2\left(\frac{3}{\sqrt{34}}\right)^2-1$$

$$=2\left(\frac{9}{34}\right)-1$$

$$=-\frac{16}{34} \text{ OR } -\frac{8}{17}$$

Q5.3.1

$$LHS=\sin(A+B)-\sin(A-B)$$

$$=\sin A.\cos B+\cos A.\sin B-(\sin A.\cos B-\cos A.\sin B)$$

$$=\sin A.\cos B+\cos A.\sin B-\sin A.\cos B+\cos A.\sin B$$

$$=2\cos A.\sin B$$

$$=RHS$$

Q5.3.2

$$\sin 77^\circ-\sin 43^\circ=\sin(60^\circ+17^\circ)-\sin(60^\circ-17^\circ)$$

$$=2\cos 60^\circ.\sin 17^\circ$$

$$=2\times\frac{1}{2}\times\sin 17^\circ$$

$$=\sin 17^\circ$$