MONYETLA NOTES FUNCTIONS AND INVERSES 9/03/2024

EXAMPLE

Determine the equation of the inverse of each of the following functions and then sketch the function and its inverse on the same set of axes, showing the line of symmetry:

(a) $f(x) = 2^x$ (b) $g(x) = \left(\frac{1}{2}\right)^x$ (c) $h(x) = \log_3 x$

Solution

(a) For f: $y = 2^x$

For f^{-1}: $x = 2^y$

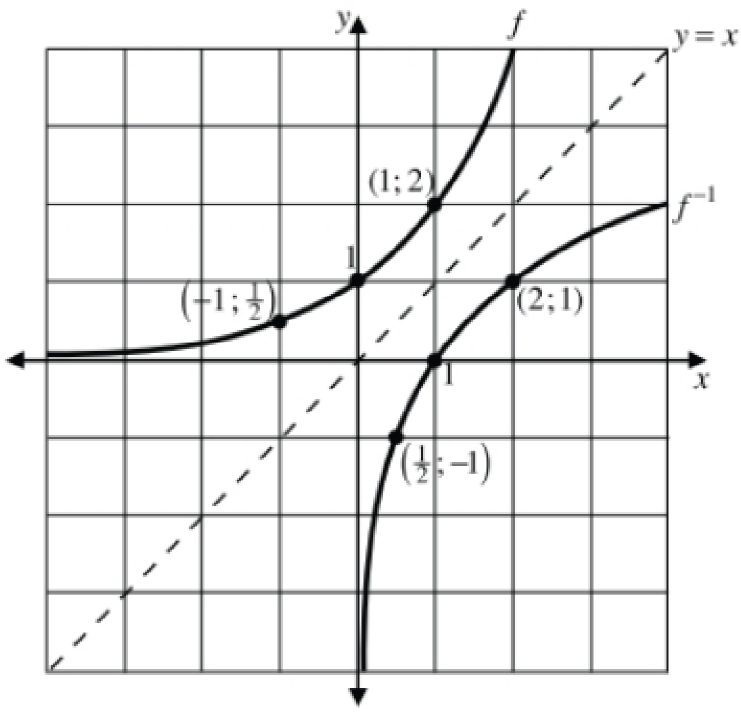
$\therefore y = \log_2 x$

$\therefore f^{-1}(x) = \log_2 x$

f is an exponential function.

The asymptote of f is $y = 0$ (the x-axis). $\therefore$ The asymptote of f^{-1} is $x = 0$ (the y-axis).

Three points on f:	**Invert** coordinates for f^{-1}:
$\left(-1; \frac{1}{2}\right)$	$\left(\frac{1}{2}; -1\right)$
$(0; 1)$	$(1; 0)$
$(1; 2)$	$(2; 1)$



(b) For g: $y = \left(\frac{1}{2}\right)^x$

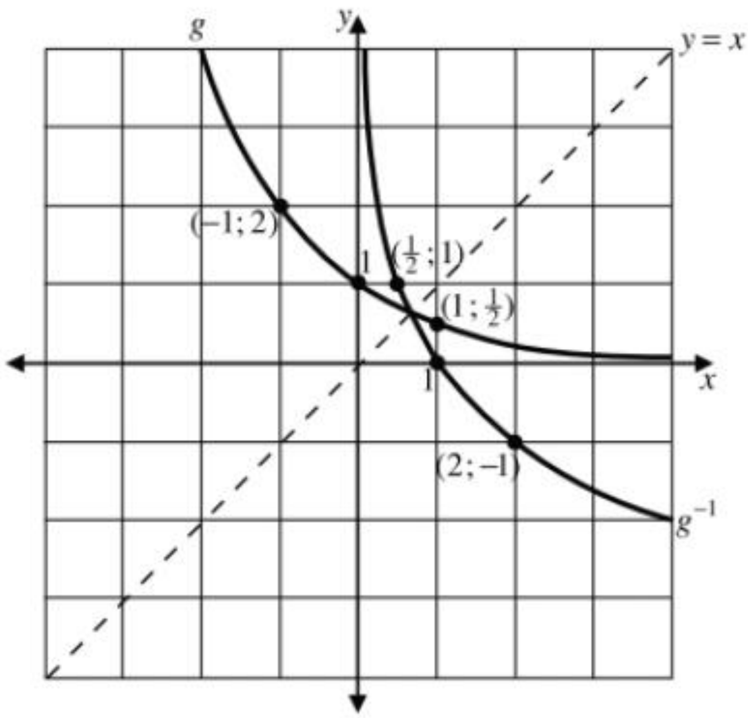
For g^{-1}: $x = \left(\frac{1}{2}\right)^y$

$\therefore y = \log_{\frac{1}{2}} x$ $\quad\quad \therefore g^{-1}(x) = \log_{\frac{1}{2}} x$

g is an exponential function.

The asymptote of g is $y = 0$ (the x-axis). $\therefore$ The asymptote of g^{-1} is $x = 0$ (the y-axis).

Three points on g:	**Invert** coordinates for g^{-1}:
$(-1;2)$	$(2;-1)$
$(0;1)$	$(1;0)$
$\left(1;\frac{1}{2}\right)$	$\left(\frac{1}{2};1\right)$



(c) For h: $y = \log_3 x$

For h^{-1}: $x = \log_3 y$

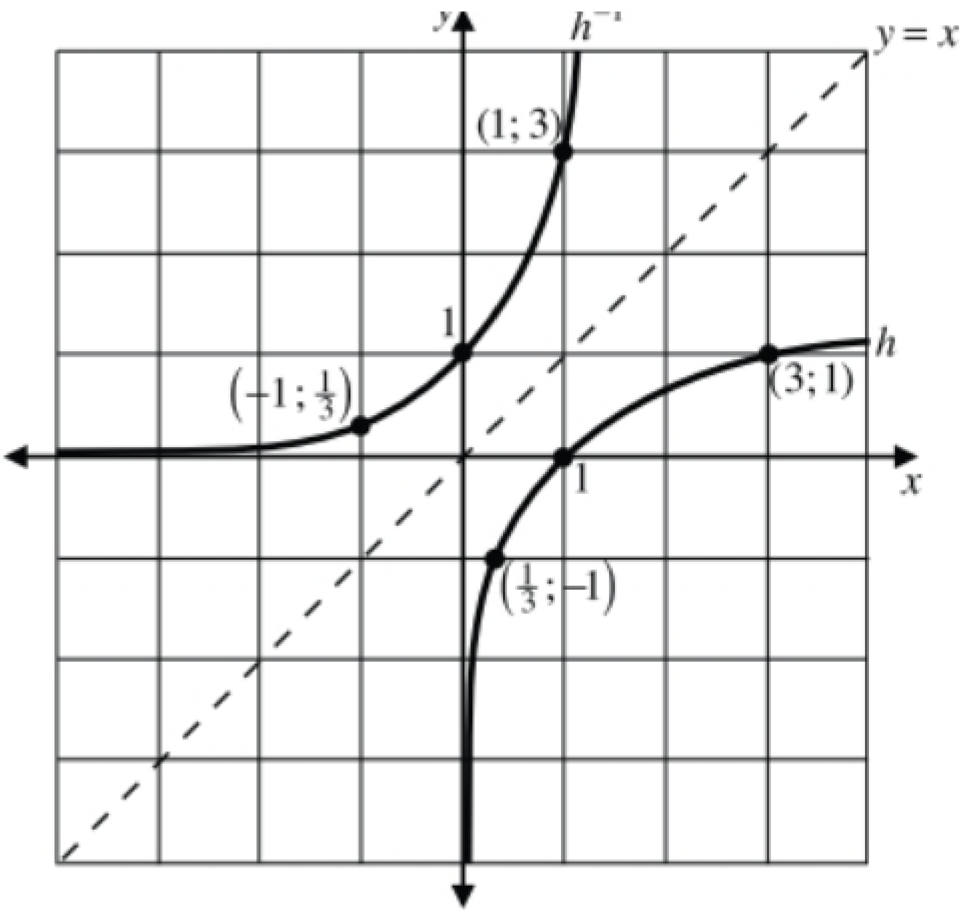
$\therefore y = 3^x$ $\therefore h^{-1}(x) = 3^x$

h^{-1} is an exponential function.

In this case it is easier to start with h^{-1}.

The asymptote of h^{-1} is $y = 0$ (the x-axis). $\therefore$ The asymptote of h is $x = 0$ (the y-axis).

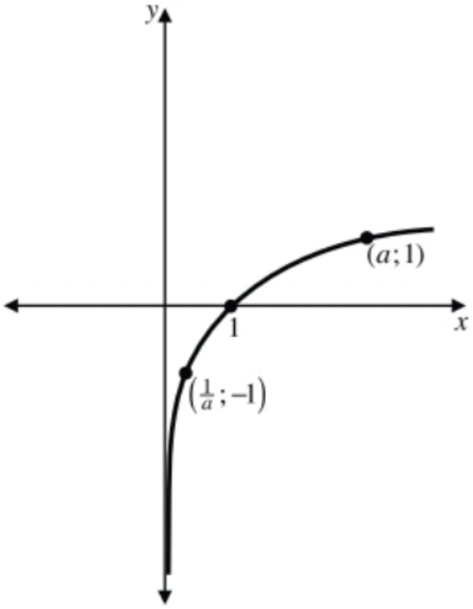
Three points on h^{-1}:	**Invert** coordinates for h:
$\left(-1\,;\frac{1}{3}\right)$	$\left(\frac{1}{3}\,;-1\right)$
$(0\,;1)$	$(1\,;0)$
$(1\,;3)$	$(3\,;1)$



THE GRAPH OF A LOGARITHMIC FUNCTION

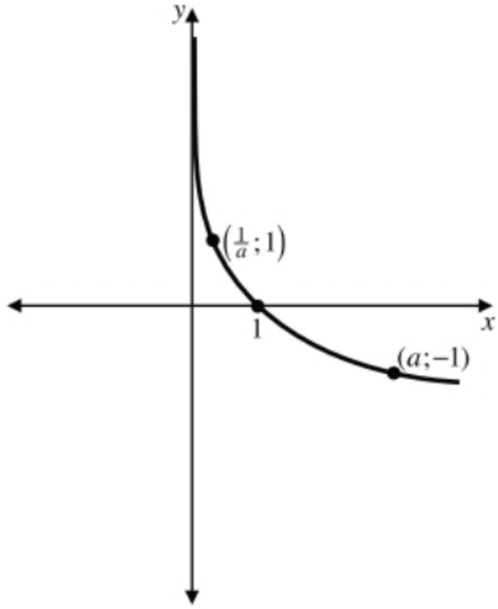
The graph of a logarithmic function can easily be drawn by considering it as the inverse of an exponential function (see Example 8 (c) above). It is, however, worthwhile to know the shapes of the two types of logarithmic functions:

$y = \log_a x \;\; ; \;\; a > 1$



- **Increasing** function
- x-intercept at $(1\,;0)$
- Asymptote: negative y-axis $(x = 0)$

$y = \log_a x \;\; ; \;\; 0 < a < 1$

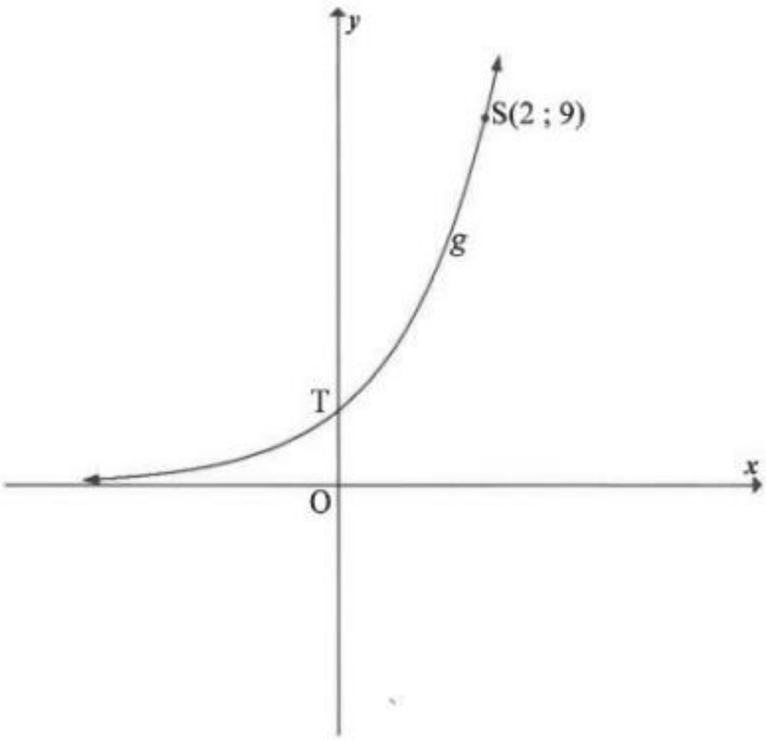


- **Decreasing** function
- x-intercept at $(1\,;0)$
- Asymptote: positive y-axis $(x = 0)$

FEB/March 2018

QUESTION 5

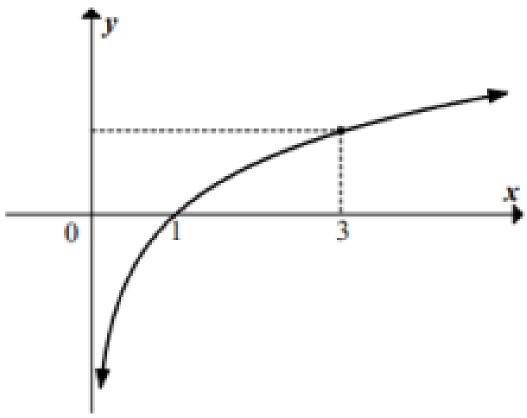
The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.



5.1	Write down the coordinates of T.	(2)
5.2	Calculate the value of a.	(2)
5.3	The graph h is obtained by reflecting g in the y-axis. Write down the equation of h.	(2)
5.4	Write down the values of x for which $0 < \log_3 x < 1$.	(2)
		[8]

FEB/MARCH 2018

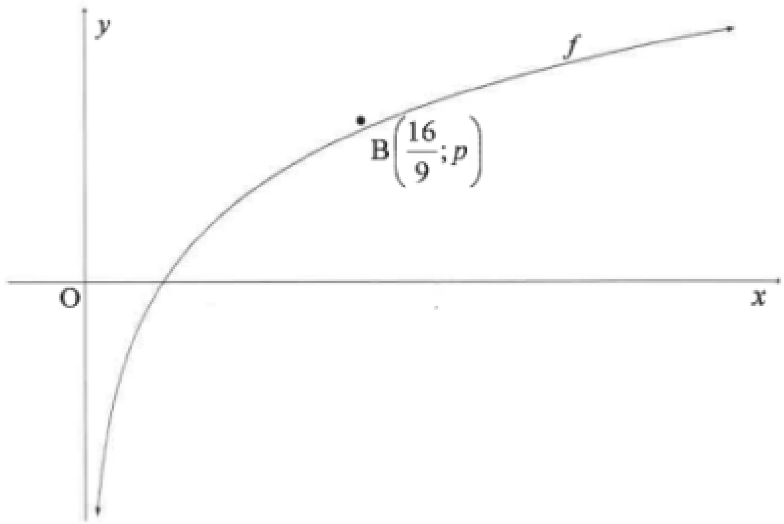
QUESTION/*VRAAG* 5

5.1	$a^0 = 1$ T(0 ; 1)	✓ $x = 0$ ✓ $y = 1$ (2)
5.2	$g(x) = a^x$ $9 = a^2$ $a = 3$ $a > 0$	✓ substitution ✓ $a = 3$ (2)
5.3	$y = \left(\frac{1}{3}\right)^x$ or $y = 3^{-x}$	✓✓ $y = \left(\frac{1}{3}\right)^x$ (2)
5.4	$3^0 < 3^{\log_3 x} < 3^1$ $1 < x < 3$ **OR**  $1 < x < 3$	✓ $1 < x$ ✓ $x < 3$ (2) ✓ $1 < x$ ✓ $x < 3$ (2) **[8]**

JUNE 2018

QUESTION 4

The graph of $f(x)=\log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9};p\right)$ is a point on f.



4.1	For which value(s) of x is $\log_{\frac{4}{3}} x \leq 0$?	(2)
4.2	Determine the value of p, without the use of a calculator.	(3)
4.3	Write down the equation of the inverse of f in the form $y = \ldots$	(2)
4.4	Write down the range of $y = f^{-1}(x)$.	(2)
4.5	The function $h(x)=\left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f. Write down the coordinates of B'', the image of B on h.	(2)
		[11]

JUNE 2018

QUESTION/*VRAAG* 4

4.1	$0 < x \leq 1$ or $(0\,;1]$	✓✓ answer (2)
4.2	$p = \log_{\frac{4}{3}} \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \left(\frac{4}{3}\right)^2$ $p = 2$	✓ substitution ✓$\left(\frac{4}{3}\right)^2$ ✓ answer (3)
4.3	$f : y = \log_{\frac{4}{3}} x$ $f^{-1} : x = \log_{\frac{4}{3}} y$ $y = \left(\frac{4}{3}\right)^x$	✓ $x = \log_{\frac{4}{3}} y$ ✓ $y = \left(\frac{4}{3}\right)^x$ (2)
4.4	$y > 0$ or $y \in (0; \infty)$	✓✓ answer (2)
4.5	$\left(-2\,; \frac{16}{9}\right)$	✓ -2 ✓ $\frac{16}{9}$ (2) [11]

WCED SEPTEMBER 2016

QUESTION 6

6.1 Given: $f(x) = 2.2^x - 1$

6.1.1 Write down the range of f. (2)

6.1.2 $g(x) = f(x-1) + 1$. Write down the equation of g^{-1}, the inverse of g in the form $y = \ldots$ (2)

WCED SEPTEMBER 2016

QUESTION/ *VRAAG* 6 (8)

#	SUGGESTED ANSWER/ *VOORGESTELDE ANTWOORD*	DESCRIPTORS/*BESKRYWERS*	Mark/ Punt
6.1.1	$y > -1; y \in R$	✓✓$y > 0; y \in R$	(2)
6.1.2	$g(x) = 2^x$ $\therefore g^{-1}: y = \log_2 x$	✓ $g(x) = 2^x$ ✓$y = \log_2 x$	(2)

NSC JUNE 2021

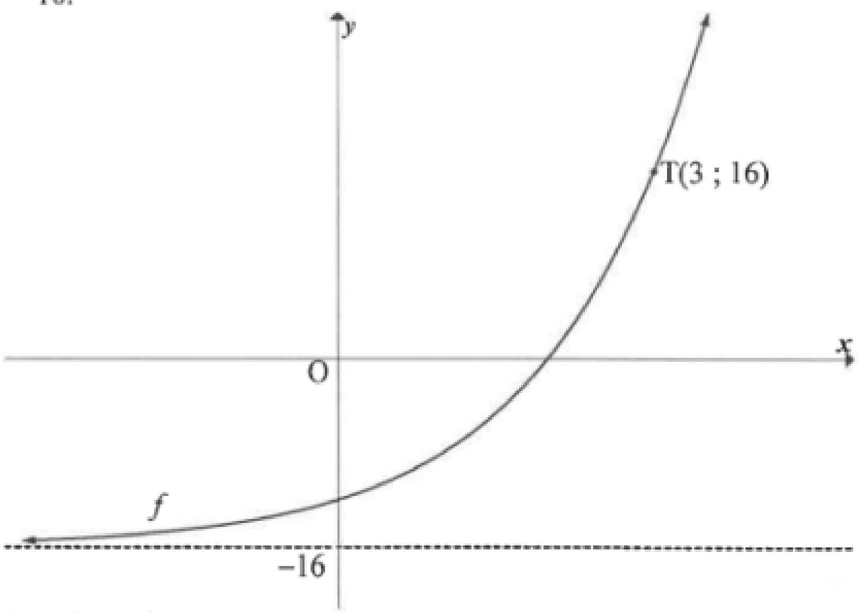
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q. (4)

[8]

NSC JUNE 2021

QUESTION/*VRAAG* 6

6.1.1	$y=3^x$ $x=3^y$ $y=\log_3 x$	✓ swop x and y ✓ equation (2)
6.1.2	$h(x)=3^{x-4}+2$ Transformation: 4 units left, 2 units down $P^/(2;9)$	✓ $x=2$ (A) ✓ $y=9$ (A) (2)
6.2	$f(x)=2^{x+p}+q$ $q=-16$ $16=2^{p+3}-16$ $2^{p+3}=32$ $2^{p+3}=2^5$ $\therefore p+3=5$ $p=2$	✓ $q=-16$ ✓ substitute (3 ; 16) ✓ $2^{p+3}=2^5$ or $p+3=\log_2 32$ ✓ $p=2$ (4)
		[8]