**Subject: Technical Mathematics**

**Paper 1 Revision Package 2020**

**Grade: 11**

Algebra
90±3

Technical Mathematics
P1
150

Financial Maths
15±3

Functions
45±3
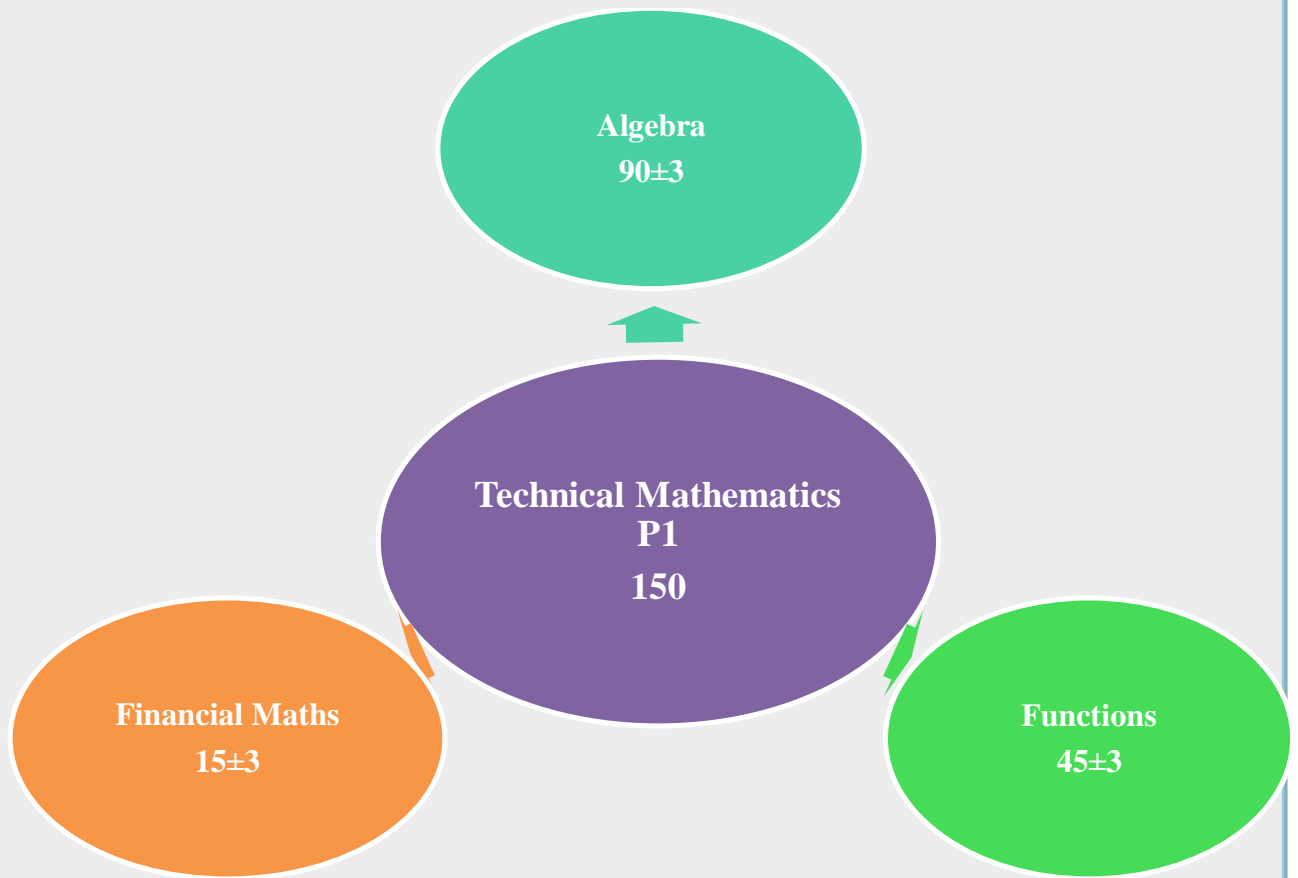
*As far as the laws of Mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.*

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**Algebra**

**QUESTION 1**

1.1     Solve for $x$:

      1.1.1       $x^2 - 5x + 6 = 0$       (2)

      1.1.2       $3x - 2 = \dfrac{4}{x}$   (Leave your answer correct to TWO decimal places.) 0       (3)

      1.1.3       $30 - x - x^2 > 0$       (4)

      1.1.4       $\sqrt{5 - x} = x + 1$       (4)

      1.1.5       $2^{2x} - 6.2^{x-1} + 2 = 0$       (3)

1.2     Solve for $x$ and $y$ simultaneously:

      $x + y = 3$       (6)

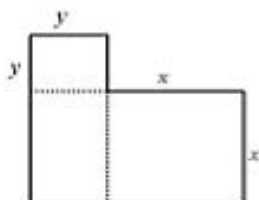      $2x^2 + 2y^2 = 5xy$

      **[22]**

1.1     Solve for $x$:

    1.1.1     $-2x(x+a)(3-x)=0$ (3)

    1.1.2     $2x=6-x^2$  (correct to TWO decimal places) (4)

    1.1.3     $5x(x-3) \leq 0$  and then represent the solution on a number line (3)

1.2     The total area represented by the L-shaped diagram below is 21 units$^2$. The equation $y-2x=-7$  represents the relationship of the sides of the two squares.



    Solve for $x$ and $y$ (dimensions of the two squares) if:
        $y-2x=-7$   and    $x^2+xy+y^2=21$ (7)

1.3     The formula below represents the moment of inertia $(E)$, with mass $(M)$ and length $(L)$:

$$E = \frac{1}{12}ML^2$$

    1.3.1     Make $L$ the subject of the formula. (2)

    1.3.2     Calculate the value of $L$, if $E = 8,3 \times 10^{-2}$ kg.m$^2$  and $M = 1,6 \times 10^3$ kg. (2)

1.4     Express 36 as a binary number. (2)

1.1     Evaluate:

    $111010_2 - 10101_2$ (2)

1.2     Solve for $x$:

    1.2.1     $x(x-3)=0$ (2)

    1.2.2     $x^2+3x+1=0$ (correct to ONE decimal) (4)

    1.2.3     $x^2+2<-3x$ (4)

1.3     Solve for $x$ and $y$ simultaneously:

    $y=x^2-1$ and $y=x+1$ (5)

1.4     Given : $x^2+bx+4=0$

    Determine the value(s) of $b$ for which the roots of the equation will be equal. (4)
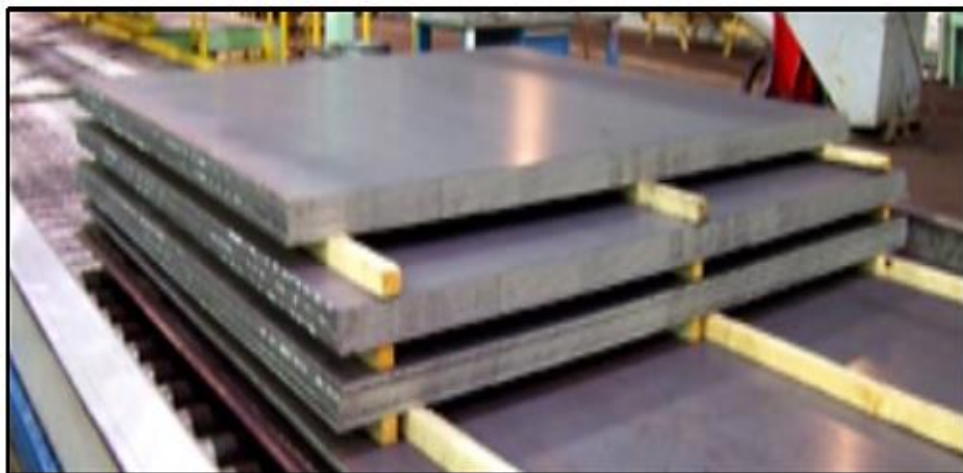
1.1 Solve for $x$:

    1.1.1    $2(7x-1)(x+2)=0$        (2)

    1.1.2    $(x-2)(3x-1)=1$ (correct to TWO decimal places)        (4)

    1.1.3    $-x^2-4x+5\geq 0$        (3)

1.2 The picture below shows packaged metal sheets by a metal sheet manufacturing company.



Each pack contains 300 metal sheets and has a thickness of 151 mm.

Determine the thickness (in metres) of one metal sheet in **Scientific Notation**.        (3)

1.3 Given:      $2y+6x=4$    and    $y^2-25x^2=4$

    1.3.1    Factorise $y^2-25x^2$        (1)

    1.3.2    Hence or otherwise, solve for $x$ and $y$ simultaneously        (6)

1.4 Given: $M=\sqrt{\dfrac{5-p}{2}}$ ;    $p\in\{1;2;3;4;5;6\}$

Determine the:

    1.4.1    Value(s) of $p$ for which M will be real        (2)

    1.4.2    Value(s) of $p$ for which M will be non-real        (1)

1.5 Describe the nature of the roots of $ax^2+bx+c=0$ if $a<0,\ b>0$ and $c=0$.        (2)

1.6 Write 86 as a binary number.        (2)

2.1    Given the roots: $x = \dfrac{-8 \pm \sqrt{q-3}}{2}$

Describe the nature of the roots if:

2.1.1    $q = 5$          (1)

2.1.2    $q = 3$          (1)

2.1.3    $q < 0$          (1)

2.2    Determine for which value(s) of $p$ will the equation $3x^2 + 7x = 2x + p$ have non-real roots.          (4)

2    Given $2x^2 + kx + 2 = 0$

     2.1    Write $\Delta$ (discriminant), in terms of $k$.          (2)

     2.2    Determine the value(s) of $k$ for which the roots of the equation will be real and equal.          (4)

2.1    Discuss the nature of the roots of $ax^2 + bx + c = 0$ for each of the following values of $b^2 - 4ac$

2.1.1    $b^2 - 4ac = -6$          (1)

2.1.2    $b^2 - 4ac = 13$          (1)

2.1.3    $b^2 - 4ac = (0)(m-1)^2$, if $m \in R$          (1)

2.2    Given : $x^2 + px + 1 = \dfrac{-p^2}{4}$

2.2.1    Write the equation in the form $ax^2 + bx + c = 0$          (2)

2.2.2    Discuss the nature of roots of the given equation          (3)

2.1    Given:      $M = \sqrt{\dfrac{k-4}{32}}$

Determine a positive integer value for $k$, for which M is:

2.1.1    non real.          (2)

2.1.2    a non –zero rational number.          (1)

2.2    Consider the equation    $mx^2 - 12x + 9 = 0$.

2.2.1    Determine the value of m if the equation has equal roots.          (4)

1.1     Solve for $x$ if:

      1.1.1    $(2x - 1)(x + 3) = 0$                                                                    (2)

      1.1.2    $x^2 - 4x - 12 > 0$                                                                    (4)

1.2     Solve for $x$ and $y$ simultaneously if:

      $x + 2y = 3$ and $x^2 - 2xy + y^2 = 9$                                                    (7)

1.3     The formula for calculating the distance travelled by an object at
      time $t$ seconds is:

$$s = ut + \tfrac{1}{2}at^2$$

      where $u$ is the initial velocity and $a$ is the acceleration of the object.

      1.3.1    Make $u$ the subject of the formula.                                          (2)

      1.3.2    Hence, calculate the value of $u$ if acceleration, $a = 3\,\text{m/s}^2$ and
              the distance travelled after 4 seconds is 36 m .                          (2)

1.4     Write 46 as a binary number.                                                              (2)

1.1     Solve for $x$ if:

      1.1.1    $x(x - 3) = 0$                                                                         (2)

      1.1.2    $3x^2 - 2x - 10 = 0$ (correct to ONE decimal place)                      (4)

      1.1.3    $2x^2 - 7x + 3 \geq 0$                                                                (3)

1.2     Determine the value (**in scientific notation**) of $\dfrac{x^2 - 4}{x + 2}$ if
      $x = 2\,000\,000\,000\,002$. (Show all your calculations)                                  (3)

1.3     Solve for $x$ and $y$ simultaneously:
      $2y + x = 3$   and $y = x^2 - x$                                                          (6)

1.4     The graph defined by $f(x) = 2x^2 - 3x + p$ is drawn below, $p$ is the $y$-intercept of $f$.

**QUESTION 2**

2.1     If 2 is a root of $2x^2 + kx = -6$, find the value of $k$.       (2)

2.2     Given: $x^2 + m + x = 3x - 2$

Determine the maximum value of $m$ for which the roots will be real.       (4)

**[06]**

**QUESTION 1**

1.1    Solve for $x$:

     1.1.1    $x(3 - x) = 0$       (2)

     1.1.2    $x^2 - 4 > 0$       (4)

     1.1.3    $\left(\dfrac{1}{27}\right)^x = 3^{-1}$       (3)

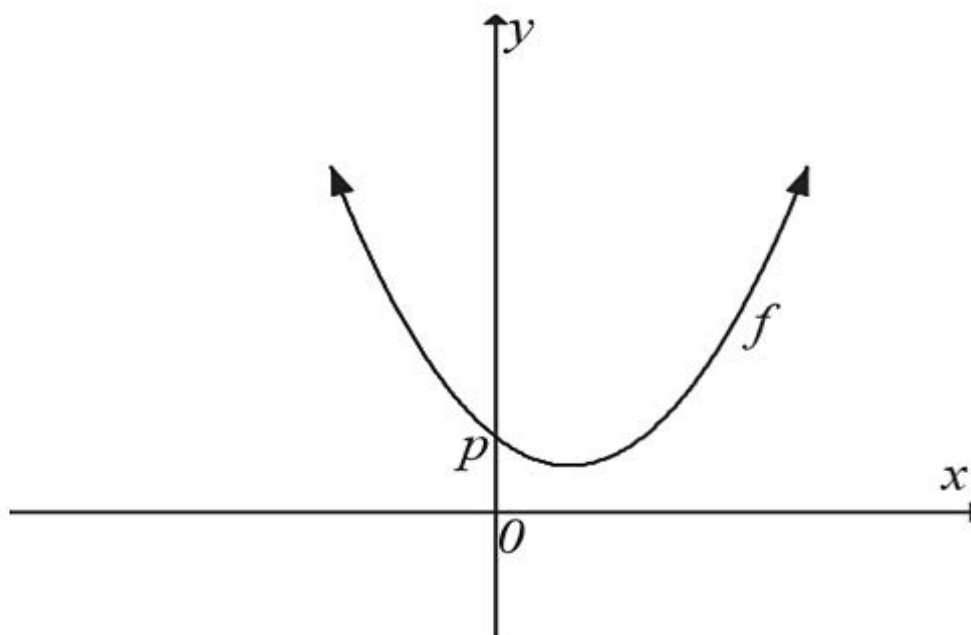1.2    Solve for $x$ and $y$ if:
$$x - y = 2$$
$$x^2 + y^2 = 10$$
      (7)

1.3    The pressure , $P(N/m^2)$, of a closed volume of gas and its temperature, $T^\circ C$, are connected by a law of the form, $P = aT + b$, where $a$ and $b$ are constants.

     1.3.1    Make **a** the subject of the formula.       (2)

     1.3.2    At $10^\circ C$, the pressure is $28\,N/m^2$ and $b = 27,25$.
            Calculate the value of $a$.       (2)

**[20]**

1.4 The graph defined by $f(x) = 2x^2 - 3x + p$ is drawn below, $p$ is the $y$-intercept of $f$.



Determine the value of $p$ for which the graph will always be above the $x$-axis. (3)

1.1 Solve for $x$:

1.1.1 $x(x^2 - 1) = 0$ (4)

1.1.2 $-1 + 5x = 2x^2$ (Correct to TWO decimal places). (4)

1.1.3 $2x^2 - 11x > 6$ and then represent the solution on a number line (6)

1.2 Solve simultaneously for $x$ and $y$:
$$y - 2x - 3 = 0 \quad \text{and} \quad x^2 - 4xy + 3y^2 = 0$$
(6)

1.3 Express the following binary number $1001011_2$ as a decimal number (2)

1.4 A sand dam has two intake pipes, one small pipe and one large pipe. A small pipe takes thirty minutes longer than the large pipe to fill the dam. Together the large and the small pipe takes 30 minutes to fill the dam. Calculate the time taken by each pipe to fill the dam. (7)

1.1      Given: $f(x) = (x - 4)(2x + 3)$

       Solve for $x$ if:

       1.1.1      $f(x) = 0$                                                    (2)

       1.1.2      $f(x) < 0$      and represent the solution on a number line      (4)

1.2      Solve for $x$ if $2x^2 - 7 = -3x$ (rounded off to TWO decimal places)      (4)

1.3      Solve algebraically for $x$ and $y$ simultaneously if:

                                                                 (6)

$$y - x + 1 = 0 \text{ and } y + 7 = x^2 + 2x$$

1.4      The FORMULA to determine the resistance in a circuit is:

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

       1.4.1      Express $R$ as the subject of the formula.      (3)

       1.4.2      Calculate $R$ the total resistance of the electrical circuit if
                   $r_1 = 30 \text{ and } r_2 = 30$.      (2)

       1.4.3      Express 15 as a binary number.      (2)

1.5      Write the value of 0,000321 in scientific notation without any rounding.      (2)

3.1      Simplify (showing ALL calculations) the following without the use of a calculator:

       3.1.1      $\left( 2a^{\frac{7}{3}} \right)^3$      (2)

       3.1.2      $\log_p p + \log_m 1$      (2)

       3.1.3      $\dfrac{\sqrt{48} - \sqrt{12}}{2\sqrt{75}}$      (3)

3.2      Solve for $x$: $\log_2(x + 62) - \log_2 x = 5$      (4)

3.1    Simplify the following without the use of a calculator. (Show all steps)

    3.1.1    $\dfrac{6.3^{2y} - 3^{2y+1}}{3^{2y}}$    (3)

    3.1.2    $\sqrt{75} - (\sqrt{64 - 16})$    (3)

    3.1.3    $\log_4 16 \times \log_3 27$    (3)

3.2    Solve for $x$:    (3)

$$2\log(x - 2) = 6$$

**[22]**

## QUESTION 3

3.1    Given $f(x) = 3x^3 - 7x^2 + 4$

    3.1.1    Determine the value of $f(2)$    (2)

    3.1.2    Hence, or otherwise, solve for $x$ if $f(x) = 0$    (5)

3.2    Determine the value of $c$, given that $g(x) = -8cx^3 + 4x^2 - 6x + 5$
leaves a remainder of 10 when divided by $x+1$    (5)
**[12]**

2.1    Simplify the following fully : $\dfrac{2^{x+1} - 2^{x-1}}{6^x}$    (4)

2.2    Prove that : $\dfrac{log_a 25 - log_a 125}{2[log_a 5^4 - log_a 5^6]} = \dfrac{1}{4}$    (5)

2.3    A rapidly growing population of rabbits on Robben Island consists of $1\,000 \times 2^{0,05t}$
rabbits after $t$ days.

    2.3.1    Approximately how many rabbits were there after 30 days?    (2)

    2.3.2    How long will it take for the rabbit population to reach 8 000?    (3)

3.1    Simplify the following without the use of a calculator (show all the steps):
$$\dfrac{5 \cdot 3^{n+2} + 6 \cdot 3^n}{3^{n+1}}$$    (3)

3.2    Solve for $x$ if: $\log_5(x + 3) + \log_5(x - 1) = 1$    (5)

2.1    Simplify the following without using a calculator:

     2.1.1     $3^n.3^4$, to a single base term           (1)

     2.1.2     $\dfrac{7.3^{n+2}}{3^{n+4}-6.3^{n+1}}$           (2)

     2.1.3     $\sqrt{32}-\sqrt{72}+\sqrt{18}$           (2)

     2.1.4     $-\log_3 243+\log_3 1$           (2)

2.2    Solve for $x$:

     2.2.1     $\dfrac{\left(4^x\right)^{2x}\cdot\sqrt{16^{-3}}}{4^x}=\left(4^x\right)^0$           (6)

     2.2.2     $x=\dfrac{\log 6-\log 2}{\log 9\left(2\log 5+\log 4\right)}$           (4)

## QUESTION 1

1.1    Simplify the following WITHOUT the use of a calculator:

     1.1.1     $2\left(\dfrac{1}{5}\right)^{-3}$           (3)

     1.1.2     $\dfrac{(\sqrt{3}-\sqrt{6})(\sqrt{3}+\sqrt{6})}{\sqrt{49}}$           (4)

     1.1.3     $\dfrac{\sqrt[3]{8x^3}}{\sqrt{9x^2+16x^2}}$           (5)

     1.1.4     $\dfrac{9^{2x-1}.12^{x+1}}{36^x.8^{1-x}}$           (5)

     1.1.5     $\dfrac{5^{2-m}-4.5^{-m}}{5^{-m}+2.5^{-m+1}}$           (4)

     1.1.6     $2\log_2 4+\log_2 10-\log_2 5$           (4)

     1.1.7     $\left(\dfrac{1}{100\,001}+\dfrac{1}{99\,999}\right)\div\left(\dfrac{1}{100\,001}-\dfrac{1}{99\,999}\right)$           (3)

1.2          Write $3\log a-\log b+4\log c$ as a single logarithm.           (3)

1.3          Prove that $\log_9 81+\log_9 1+\log_2 16-\log_{25} 0,04=5$           (5)

                                                                  **[36]**

2.1    Simplify the following WITHOUT using a calculator:

2.1.1    $\dfrac{\sqrt{-18} \cdot \sqrt{-12}}{\sqrt{-6}}$        (3)

2.1.2    $\log 6 + 2\log 20 - \log 3 - 3\log 2$        (5)

2.2

2.2.1    Show that $\sqrt{\dfrac{5^{x+1} - 5^{x}}{5^{x-1}} + 5} = 5$        (4)

2.2.2    Hence, solve for $x$ if $\sqrt{\dfrac{5^{x+1} - 5^{x}}{5^{x-1}} + 5} = \left(\dfrac{1}{5}\right)^{x-2}$        (4)

2.2.3    Solve for $x$ if $4\log_{2} x - 1 = \log_{2} 8$        (4)

3.1    Simplify ( showing ALL calculations)the following without the use of the calculator:

3.1.1    $\left(5^{\frac{1}{2}} m^{\frac{1}{2}}\right)^{4}$        (2)

3.1.2    $\log 10 + \log_{5} 1$        (2)

3.1.3    Solve for $x$:   $\log_{3}(3x+2) + \log_{3} 2 = 2$        (3)

| I can | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Solve equations using: | Factorisation | | | | | | | |
| | Quadratic formula | | | | | | | |
| | Simultaneous methods | | | | | | | |
| Solve and make a variable the subject of the formula | | | | | | | | |
| Convert decimal numbers to binary numbers and vice versa | | | | | | | | |
| Determine the nature of the roots and interpret them | | | | | | | | |
| Simplify | Logarithms | | | | | | | |
| | Exponents | | | | | | | |
| Solve | Logarithms Equations | | | | | | | |
| | Exponential Equations | | | | | | | |
| Total | | | | | 90 | | | |

**Functions**

## QUESTION 5

The external structural design of a newly erected bridge represents a parabolic function defined by $h(x) = -(x-3)^2 + 4$ and a linear function defined by $g(x) = -x + 5$ that represents a steel cable supporting one side of the parabolic structure.

The parabolic structure touches the horizontal roadside at A and B.

Point B and C are points of contact of $h$ and $g$.

The vertical steel cable DE touches the horizontal roadside at E and cuts CB at F.

D is the maximum point of the bridge.

Two supportive vertical beams are at each edge of the bridge which are each 1 unit away from the point of contact between the bridge structure and the road surface.
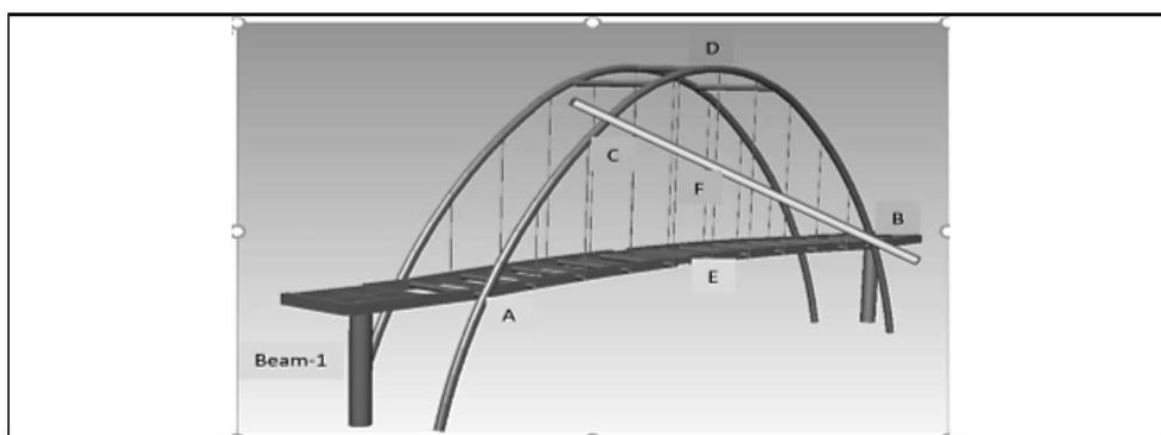


[Source: www.fbridgeworkoutpics.blogspot.com]

The following cartesian diagram models the above design:
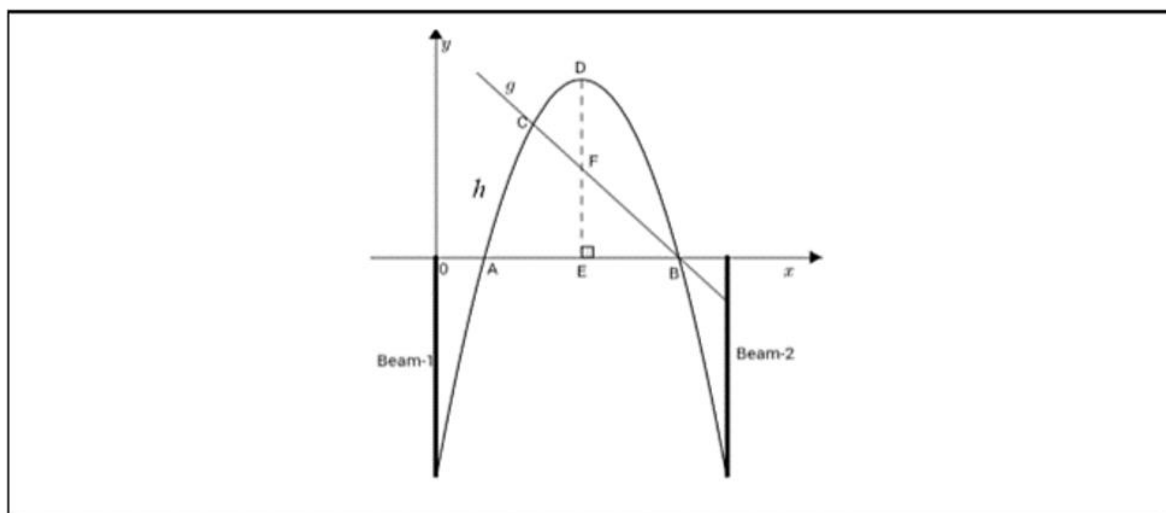


5.1 Determine the:

    5.1.1    Coordinates of A and B, the $x$-intercepts of $h$    (4)

    5.1.2    Coordinates of D, the turning point $h$    (2)

5.1.3    Domain of $h$                               (3)

5.1.4    Maximum height of the bridge above the road       (1)

5.1.5    Height of the beams, if the heights of the beams are the same       (2)

5.1.6    Range of $h$                                 (2)

5.2    A truck driver driving a truck of 4,5 metres high wishes to use the bridge as it shortens the distance he must travel.

      Will the truck be able to go over the bridge?
      Justify your answer by means of correct mathematical reasoning.       (3)

5.3    Calculate the length of FD.       (3)

Given:

$$f(x) = -(x - 2)^2 + 4$$

7.1    Determine the $x$-intercepts of $f$.       (3)

7.2    Write down the $y$-intercept of $f$.       (1)

7.3    Write down the coordinates of the turning point of $f$.       (2)

7.4    On the DIAGRAM SHEET provided, sketch the graph of $f$. Clearly show all the intercepts with the axis and the turning point of the graph.       (4)

7.5    Write down the range of $f$.       (1)

7.6    Determine the coordinates of the turning point of $g$ if $g(x)$ is the result of shifting $f(x)$, 1 unit to the right and 1 unit downwards.       (2)

4.1 The graphs below represent $h(x) = -x^2 - 3x + 10$ and $g(x) = -mx + 10$. The two graphs intersect at points A and C. The points A, B and C are intercepts with the axes of $h$. D is the turning point of $h$.
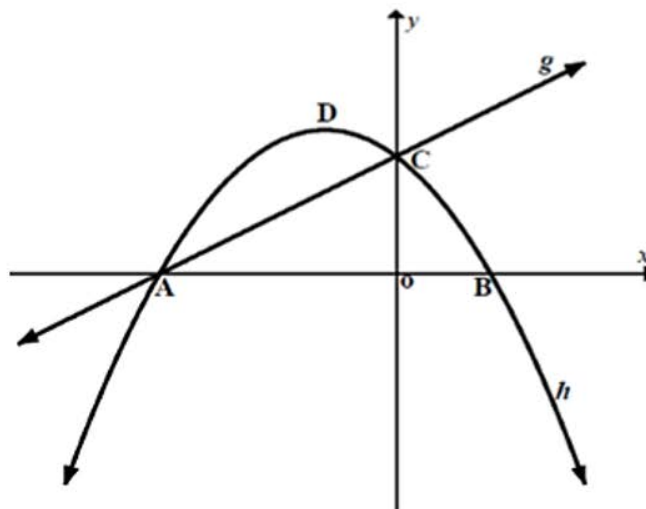


Determine the following:

4.1.1 The co-ordinates of A and B.                                 (4)

4.1.2 The co-ordinates of C.                                      (2)

4.1.3 The co-ordinates of the turning point D.                 (3)

4.1.4 The value of $m$.                                            (2)

4.1.5 The value(s) of $x$ for which $h(x) \times g(x) < 0$              (2)

4.3    In the sketch below is the graphs of a parabola $f(x) = a(x - p)^2 + q$ and a hyperbola $g(x) = \dfrac{k}{x} + n$. A $(1; 4)$ is the turning point of the parabola which intersect the $x$-axis at **B** and **C**. The $y$-intercept of the parabola is **E**. **D** is the $x$-intercept of the hyperbola.
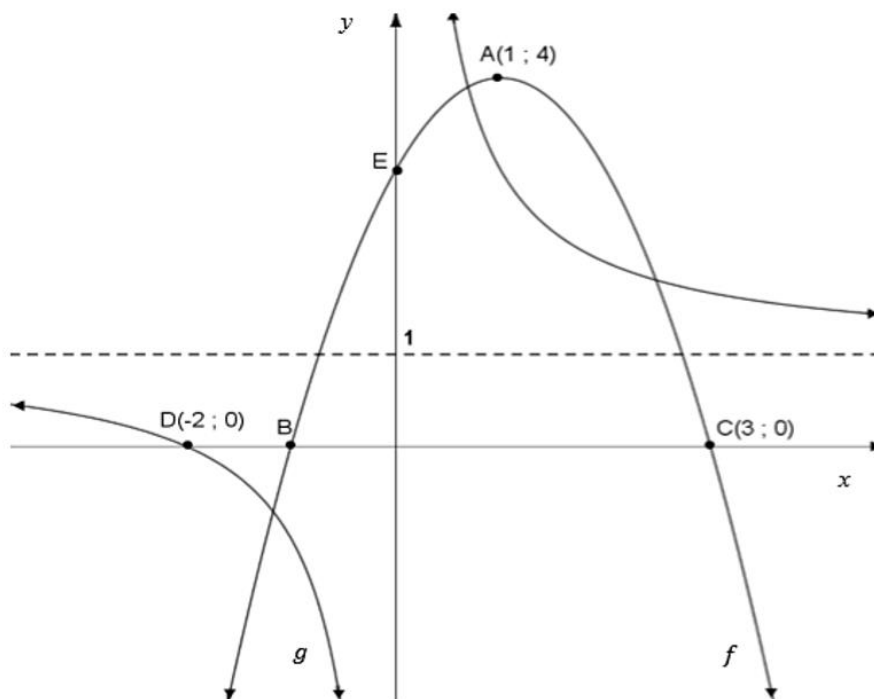


4.3.1    Write down the coordinates of B.      (2)

4.3.2    Write down the value(s) of $p, q$ and $n$.      (3)

4.3.3    Determine the equation of $f$.      (3)

4.3.4    Give the equation of the axes of symmetry for $f$?      (1)

4.3.5    Determine the equation of $g$.      (3)

4.3.6    Write down the value(s) of $x$, for which $f(x).g(x) < 0$.      (2)

4.4    Thabo draws a circle with a compass. He measures the diameter of the circle and gets a distance of 26cm. Thabo needs your help to write down an equation for the graph.      (3)

4.1     Given: $g(x) = 2^{-x} - 1$ and $h(x) = -\dfrac{6}{x} - 1$

> 4.1.1     Write down the equations of the asymptotes of $h$.                          (2)
>
> 4.1.2     Determine the coordinates of the $x$-intercept of $h$.                      (2)
>
> 4.1.3     Sketch the graphs of $g$ and $h$ on the same set of axes on the ANSWER
> SHEET provided. Clearly show the asymptotes and the intercepts with the
> axes.                                                                                 (5)
>
> 4.1.4     Show that $(-2 ; 3)$ is a point on the graph of $g$.                        (1)
>
> 4.1.5     Write down the range of $g$.                                                (1)
>
> 4.1.6     Write down the domain of $h$.                                               (1)

4.1     Given :                         $f(x) = 2^x - 1$

> 4.1.1     Sketch the graph of $f(x)$ on the ANSWER sheet provided. Clearly show
> all the intercepts with the axes and the asymptote.                                   (3)
>
> 4.1.2     Determine the domain and the range of $f(x)$.                               (3)
>
> 4.1.3     Write down a new equation $g(x)$ if the graph of $f(x)$ move 5 units up.    (1)

## QUESTION 6

Given the functions defined by : $f(x) = 2^x$ and $g(x) = \dfrac{-2}{x} + 1$

6.1     Determine the:

> 6.1.1     Coordinates of the $x$- intercept of $g$                                    (2)
>
> 6.1.2     $y$- intercept of $f$                                                       (1)
>
> 6.1.3     Equations of asymptotes of $f$ and $g$                                      (2)

6.2     On the same set of axes, sketch the graphs of $f$ and $g$ on the DIAGRAM SHEET
provided. Indicate all the intercepts, asymptotes and directions of the two graphs.     (7)

6.3     Determine the:

> 6.3.1     Domain of $g$                                                               (2)
>
> 6.3.2     Values of $x$ for which $f(x) > g(x)$                                       (1)

4.2   Given: $f(x) = \dfrac{2}{x} + 1$ and $g(x) = 2^x + 1$

     4.2.1   Write down the equations of the asymptotes of $f$.        (2)
     4.2.2   Calculate the coordinates of the $x$ intercept of $f$.        (2)
     4.2.3   Sketch $f$ and $g$ on the provided grid. Show all intercepts with axes and asymptotes.        (6)
     4.2.4   Write down the range of $g$.        (2)
     4.2.5   State all values of $x$ for which $f(x) \geq g(x)$        (2)

The graph defined by $f(x) = \dfrac{a}{x} + q$ is drawn below. The graph cuts the $x$- axis at $-2$
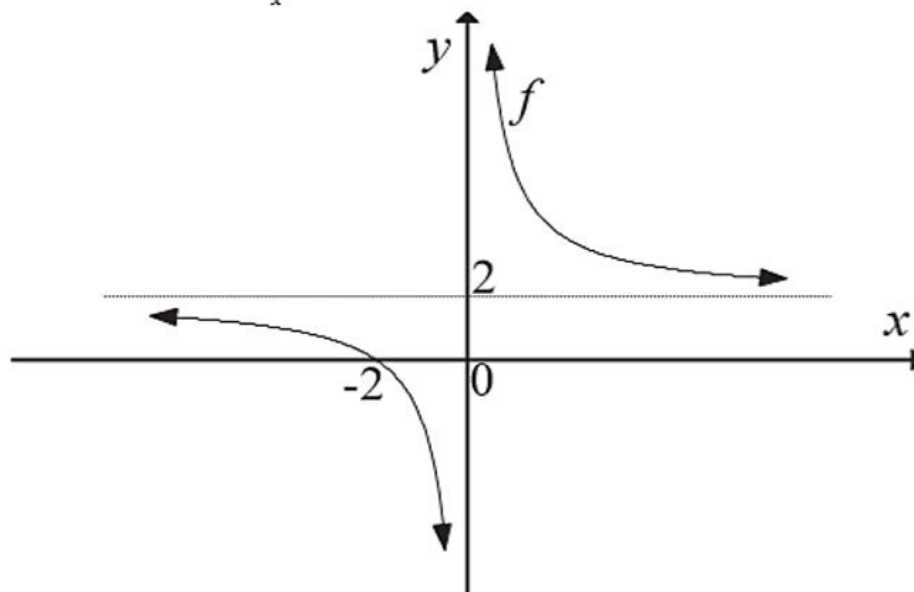


Determine:

5.1     The numerical value of $q$        (1)

5.2     The numerical value of $a$        (2)

5.3     The equations of asymptotes of $f$        (2)

5.4     The domain of $f$        (2)

Given: $g(x) = \left(\dfrac{1}{3}\right)^x$ and $h(x) = -\dfrac{3}{x}$

3.1 Write down the equations of the asymptotes of $h$.      (2)

3.2 Determine the $y$-intercept of $g$.      (1)

3.3 Sketch the graphs of $g$ and $h$ on the same set of axes on the ANSWER SHEET provided. Clearly show the asymptotes and the intercepts with the axis.      (3)

3.4 Write down the domain of $h$.      (1)

3.5 Determine the value(s) of $x$ for which $g(x) > h(x)$.      (2)

4.1 Given the functions defined by $g(x) = \left(\dfrac{1}{2}\right)^x - 4$ and $h(x) = -x - 4$

     4.1.1 Is the $g$ an increasing or decreasing function?      (1)

     4.1.2 Write down the equation of the asymptote of $g$.      (1)

     4.1.3 Determine the $x$ and $y-$ intercepts of $g$.      (4)

     4.1.4 Determine the $x-$ intercept of $h$.      (2)

     4.1.5 Draw sketch graphs of $g$ and $h$ on the same system of axes. Indicate all the intercepts with the axes and asymptote.      (5)

4.2 Draw a sketch graph of $k(x) = \dfrac{a}{x} + q$, indicating intercept (s) and asymptote, with the following conditions:

- $a > 0$
- $x \neq 0$
- $y \neq -2$
- $k(4) = 0$      (3)

In the diagram drawn below, is a semi-circle defined by $h(x) = -\sqrt{r^2 - x^2}$ and a tangent to a semi-circle at point $P(-3; -3)$.
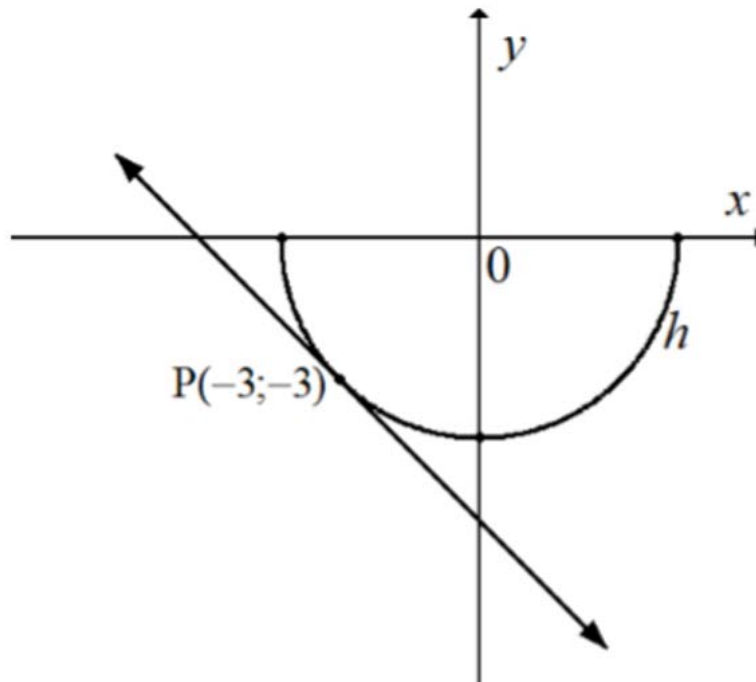


Determine the:

4.1    Equation of the semi-circle, $h$                                                (2)

4.2    Equation of the tangent to the semi-circle at P in the form $y = \ldots$      (4)

4.3    Range of $h$                                                                       (2)
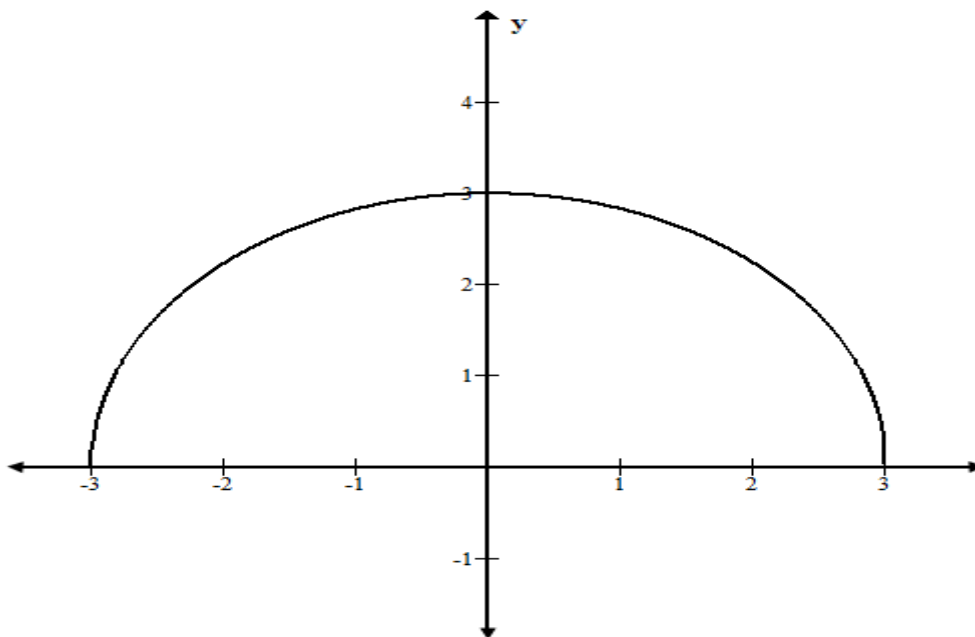
**QUESTION 6**

An engineer has designed the structure of the tunnel. The picture below represent the end product of the tunnel.



*Source: www.google.com/search*

The equation that defines the structural representation of the tunnel is given by $f(x) = \sqrt{9 - x^2}$. The sketch below represents the tunnel.



6.1    Determine the range of $f$.                                                                          (2)

6.2    Determine the radius of the semi-circle.                                              (2)

6.3    If the graph of $f$ is reflected across the $x$ −axis and the radius increases by 2 units, write the equation of the new circle in the form of $h(x) =……$      (2)

**[6]**

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The graphs of a semi-circle defined by $h(x) = +\sqrt{4 - x^2}$, centred at the origin and an exponential funtion defined by $g(x) = 2^x + q$ are drawn below :

- A and B are the $x$-intercepts of $h$
- Points $(0; 2)$ and $C\left(-\dfrac{73}{50}; k\right)$ are points of intersection of $g$ and $h$
- The horizontal asymptote of $g(x)$ cuts the $y$-axis at $q$
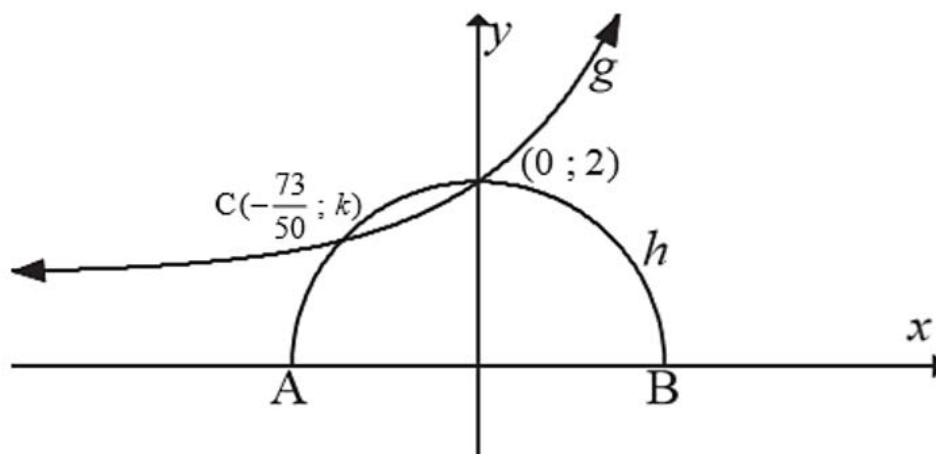


Determine:

6.1    The coordinates of A and B                              (2)

6.2    The numerical value of $q$                                 (1)

6.3    The equation of a straight line passing through B and point $(0;2)$    (2)

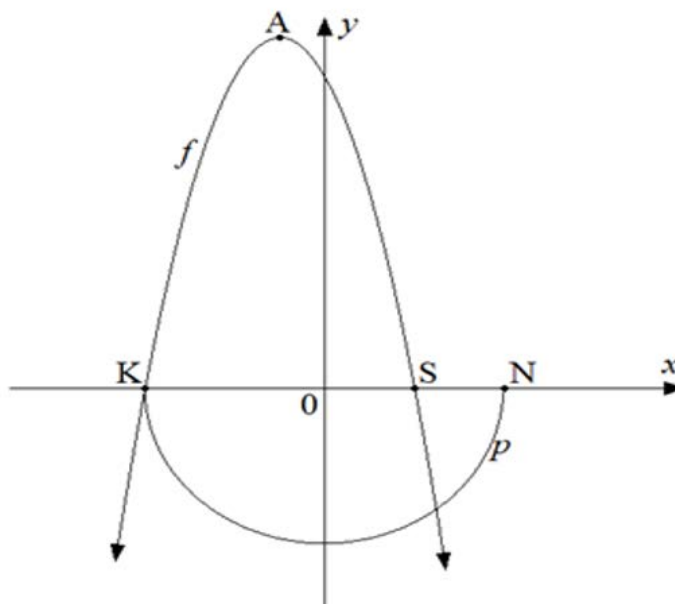6.4    The values of $x$ for which $g(x) < h(x)$                     (2)

The sketch below represents the functions defined by $f(x) = -x^2 - 2x + 8$ and
$p(x) = \sqrt{m^2 - x^2}$
S is the $x$ – intercept of $f$, while K is the $x$ – intercept of both $f$ and $p$.
A and J are the turning point and $y$ – intercept of $f$ respectively.
N is the $x$ – intercept of $p$ and C is a point on $p$.



Determine:

| 5.1 | Coordinates of K and S. | (5) |
|---|---|---|
| 5.2 | Coordinates of A. | (4) |
| 5.3 | Write down the maximum value of $f$. | (1) |
| 5.4 | Write down the range of $f$. | (2) |
| 5.5 | Equation of $p$. | (2) |
| 5.6 | Hence write down the radius of $p$ | (1) |

4.2      Sketched below are the graphs defined by $f(x) = a(x+p)^2 + q$ and $g(x) = \sqrt{36 - x^2}$ with $T(1 ; 8)$ the turning point of $f$. Line TM is drawn such that TM is perpendicular to the $x$-axis. Points L and K are the intercepts of $f$. Point L is a point of intersection of $f$ and $g$. Point R lies on both line TM and the graph of $g$.
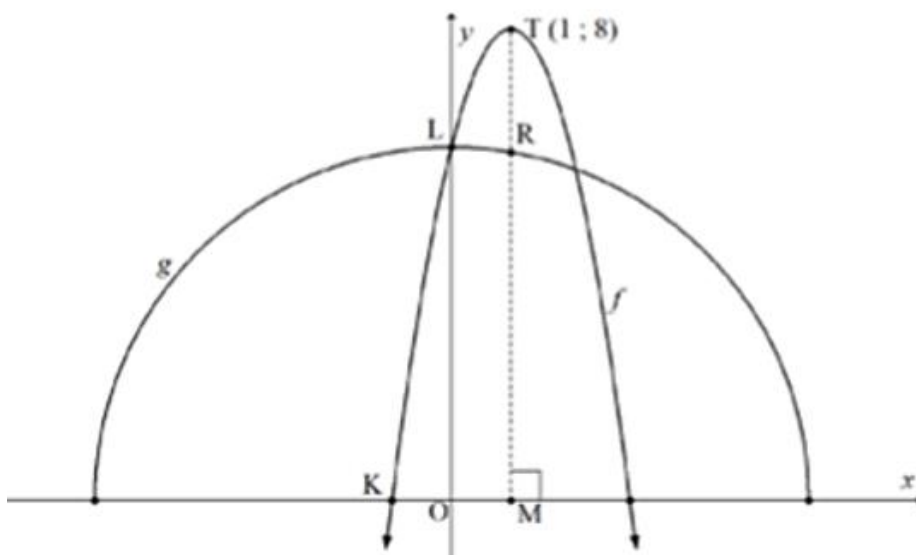


4.2.1      Write down the coordinates of M.      (1)

4.2.2      Determine the length of TR (leave your answer in surd form).      (3)

4.2.3      Show that $(0 ; 6)$ are the coordinates of L.      (1)

4.2.4      Hence, show that the graph of $f$ is defined by $f(x) = -2(x+1)(x-3)$.      (4)

4.2.5      Hence, give the coordinates of K.      (1)

4.2.6      Determine the values of $x$ for which $f(x) \times g(x) > 0$ and $x < 0$      (2)

4.2      Given the function $f(x) = x + 2\sqrt{5}$ and the equation of the circle $x^2 + y^2 = 20$

4.2.1      Write down the radius of the circle.      (1)

4.2.2      Sketch the circle, $x^2 + y^2 = 20$ on your answer-sheet.      (3)

4.2.3      Sketch the function $f$ on the same system of axes as the circle.      (2)

4.2.4      Write down the coordinates of a point where the two graphs intersect.      (1)

| I can | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| **Clearly draw/sketch the graph of** | A parabola | | | | | | |
| | A hyperbola | | | | | 26 | |
| | An Exponential and log Function | | | | | | |
| | Semi-circle | | | | | | |
| **Calculate** | Points of intersection of two graphs | | | | | | |
| **Determine the coordinates of** | The turning points of a parabola | | | | | | |
| | The $x$ and $y$ intercepts of various functions | | | | | | |
| **Determine the domain and range of various functions** | | | | | | | |
| **Determine the equation of the axis of symmetry** | | | | | | | |
| **Total** | | | | | $\dfrac{\phantom{00}}{45}$ | | |

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**Financial Maths**

5.1     The annual effective interest rate charged by a financial institution is 6,7%. Calculate the nominal interest rate charged per annum if compounded monthly.   (4)

5.2     A company bought a new 3D wheel-alignment machine for R240 000. The machine depreciated at a rate of 16% per annum to half its original value over a certain period.



5.2.1     Give the depreciated value of the machine at the end of the period.   (1)

5.2.2     Determine how long it will take for the machine to depreciate to half its original value. Give your answer to the nearest year.   (5)

5.3     Mr Bohlale invested R40 000 at a bank for 7 years. The interest rate for the first 4 years was 11,2% per annum, compounded quarterly. The interest rate then changed to 13% per annum compounded annually for the remaining years. Calculate the total amount of money that Mr Bohlale will receive at the end of the investment period.   (5)

4.1     An amount of R2 500 is deposited into a savings account at 14% interest per annum, compounded quarterly.

4.1.1     Calculate the nominal interest rate per quarter which the savings account will accumulate.   (1)

4.1.2     Determine the effective interest rate per annum, correct to one decimal.   (3)

4.1.3     Calculate the amount of money in the savings account at the end of 7 years.   (3)

4.2     The Eastern Cape Construction Consortium opened a savings account into which a sum of R250 000 was deposited. The money in the account will be used to purchase a tipper truck with an estimated value of R800 000 after 5 years.

- The account accumulates an interest rate of 8% p.a. compounded monthly, for the first two years and 10% p.a. compounded quarterly for the remaining 3 years.
- A sum of R80 000 is deposited into the account at the end of the third year.

Determine how much will be in the savings account after 5 years.   (8)

6.1    Determine the nominal interest rate compounded quarterly if the effective interest
       rate is 8% per annum.                                                                          (3)

6.2    In an experiment conducted by learners in a science class, as shown in the picture
       below, a 90 mm piece of chalk is immersed in a cup containing vinegar to test the
       effects of an acid on a piece of chalk. The chalk dissolves at a rate of 5% per minute,
       compound decrease.
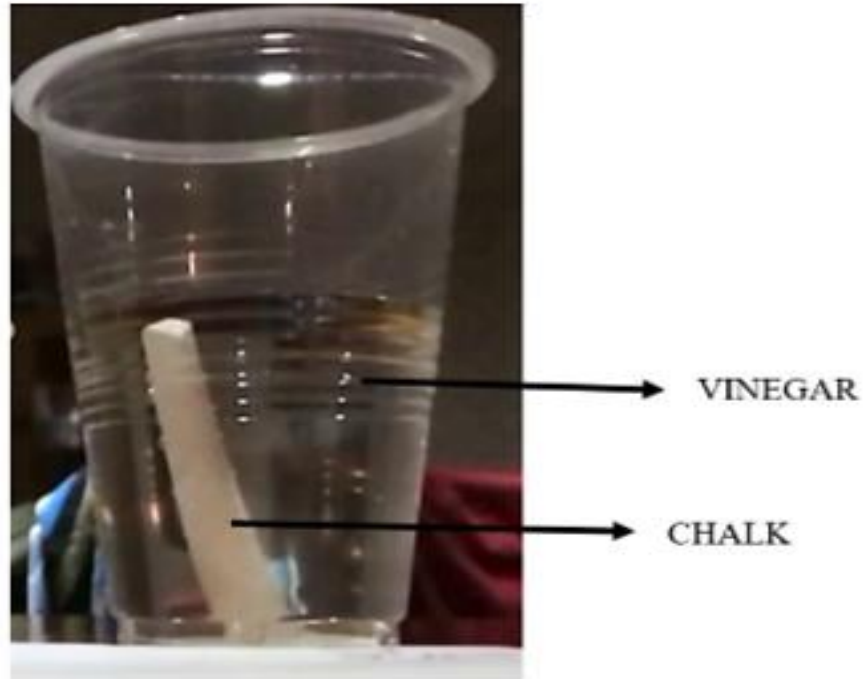


       Determine, how long (in minutes) it will take for the chalk to dissolve to half its
       original length.                                                                               (5)

6.3    R300 000 is invested at 12% per annum, compound interest for 3 years.
       After 3 years a deposit of R65 000 is made at 9% per annum compounded quarterly
       for the remaining 4 years.

       Calculate how much will be in the account at the end of 7 years.                               (6)

6.1    Nominal interest rate charged on an investment is 7,2% compounded
       quarterly. Calculate the effective interest rate of the investment.                (3)

6.2    The air pressure of the tyre deflated from 210 kPa at a depreciation
       rate of 10% kPa per minute.

       6.2.1   Is 210 kPA initial or final air pressure of the tyre?                       (1)

       6.2.2   Does the air pressure indicates growth or decay?                            (1)

       6.2.3   Determine the air pressure after 3 minutes.                                 (2)

6.3    Mr. Nale invests R 150 000 to buy a drilling machine for his company.
       Interest earned is 10,5% p.a. compounded monthly for 5 years.

       At the end of the fourth year, Mr. Nale withdraws R 50 000 from the
       investment account ( and continues with the investment.)
       How much will he receive at the end of the investment period?                       (7)

4.1    A group of learners are tasked to clean a fish pond that has been neglected wherein a
       certain type of bacteria has been developing. At the beginning of the cleaning process, a
       2-litre sample is tested and 12 000 bacteria are found. After 30 minutes, another 2-litre
       sample is tested and the bacteria count has decreased to 4 000. It is found that the bacteria
       decrease rate follows the compound decrease formula.

       4.1.1   Calculate the estimated decrease rate of the bacteria per minute.           (4)

       4.1.2   Use the results obtained in QUESTION 4.1.1 to determine how many bacteria
               will be present at the end of 1 hour.                                        (3)

4.2    Mabeka Construction Company bought a second hand TLB machine as shown below.



   •   The TLB machine cost R600 000 from a dealership.
   •   The company paid 18% deposit and took out a bank loan to pay the remaining
       amount.
   •   The bank charges 15 % interest per annum, compounded monthly.

       4.2.1   Calculate the amount the company loaned from the bank.                       (2)

       4.2.2   Determine the number of years it took Mabeka Construction Company to pay off
               the loan if the company paid R1 204 860, 32 in total.                        (4)

5.1  The nominal interest rate charged on an investment is 6, 7% compounded half yearly.          (3)
     Calculate the annual effective interest rate for the investment.

5.2  The air pressure of a punctured BMW tyre deflated from 260 kPa to 69 kPa at a decreasing     (5)
     rate of 7, 5% per minute. Determine how long it took the tyre to deflate from 260 kPa to 69
     kPa. Give your answer to the nearest minute.

5.3  Mr Mogale invested an amount of R200 000 to buy a vertical drilling machine for his
     company. Interest is calculated at 11, 5% compounded quarterly p.a. for 5 years. At the end of
     the 3$^{rd}$ year, Mr Mogale withdrew an amount of R50 000 from the investment and then
     continued investing the balance for the remaining period.



     Determine the value of the investment at the end of the investment period.                    (7)

5.1      Examine the following problems :

         5.1.1      Mnr. van Dyk invest an amount of R100 000 at the age of 30 years. He
                    hopes that his investment will increase to R140 000 after 10 years.             (3)
                    Determine the rate at which it must be invested on simple interest.

         5.1.2      Calculate the effective interest rate of a nominal rate of 12,5%
                    compounded half yearly.                                                          (3)

         5.1.3      A Mazda motor vehicle with a value of R93 250 depreciate at a rate of
                    7,5% per year on a straight line basis. What will the value of the car be        (3)
                    after 4 years?

         5.1.4      Sipho invest R5 000 at 8,7% per annum compounded quarterly. After 3
                    years Sipho adds another R2 200 to his investment, which is still calculated     (5)
                    at the same rate. What is the value of the investment after 7 years?

**QUESTION 8**

8.1　The graph below represent the price of the tractor. The tractor depreciated from R120 000 to R11 090, 91 over a period of 12 years on a reducing balance method.
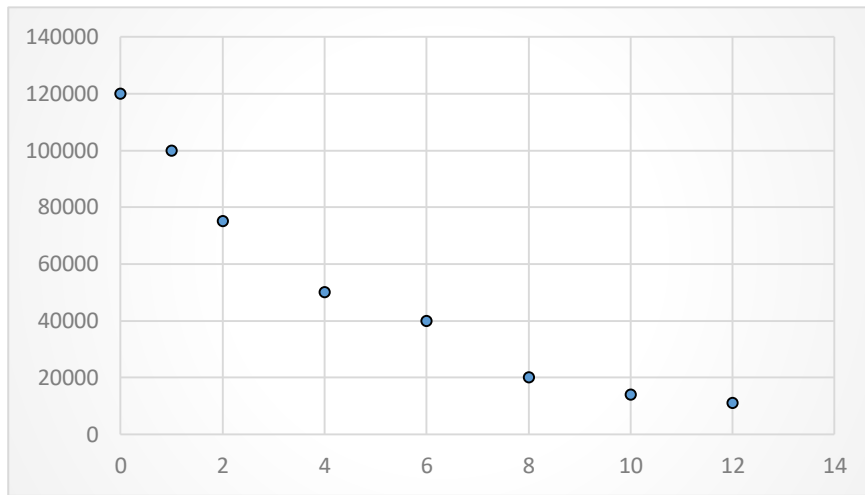


(5)

Calculate the rate of depreciation per annum. (The rate was fixed over the 12 years.)

8.2　An investment earns interest at a rate of 7% per annum, compounded semi-annually. Calculate the effective annual interest rate on this investment.

(4)

8.3　Sandile made an initial deposit of R15 000 into an account that paid interest at 9, 6% p.a, compounded quarterly. Six months later she withdrew R5 000 from the account. Two years after the initial deposit she deposited another R 3 500 into this account. How much does she have in the account 3 years after her initial deposit?

(5)

**[14]**

| I can | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Use simple and compound decay formulae: $A = (1-in)$ and $A = (1-i)$ to solve problems (including straight line depreciation and depreciation on a reducing balance). | | | | | | |
| The effect of different periods of compound growth and decay, including **nominal and effective** interest rates. | | | | | | |
| Solve problems involving present value and future value annuities. | | | | | | |
| Make use of logarithms to calculate the value of n , the time period, in the equations  and | | | | | | |
| Critically analyse investment and loan options and make informed decisions as to best option(s). | | | | | | |
| **Total** | | | | | | |
| | | | | $\overline{15}$ | | |