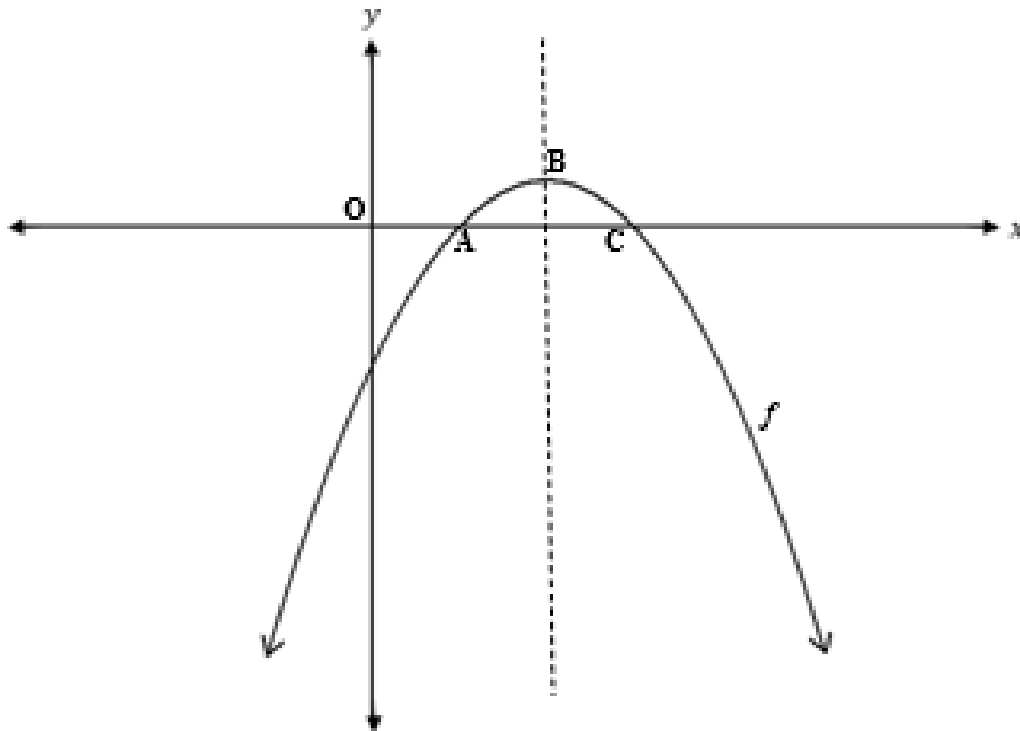# FUNCTIONS AND CALCULUS

QUESTION 5

5.1   Sketched below is the parabola f, with the equation $f(x) = -(x-2)^2 + 1$.

5.1   Write down the coordinates of B.   (1)
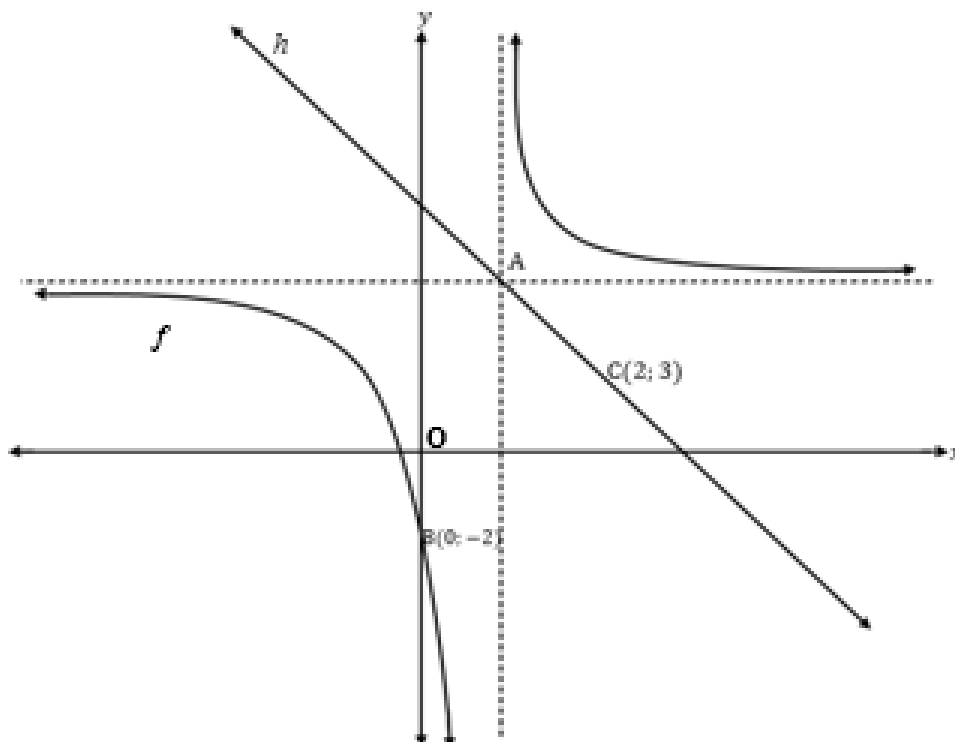
5.2   Write down the equation of the axis of symmetry.   (1)

5.3   Determine the coordinates of A and C.   (3)

5.4   For which value(s) of x will $f(x) \leq 0$.   (2)

5.5   Determine the average gradient between A and B.   (2)

5.6   The graph of $g(x)$ is obtained by shifting the graph of f, 2 units
        to right, 1 unit downwards, write down the equation of $g(x)$, and
        then sketch the graph of $g(x)$ on the same set of axis.   (3)

[12]

QUESTION 6

In the sketch below the graph of $f(x) = \dfrac{a}{x+p} + 4$ is given. The asymptotes of f intersect at point A. The graph of f cuts the y-intercept at $B(0;-2)$. The axis of symmetry of f, is the line h. Point C coordinates $C(2;3)$ is the point on h.
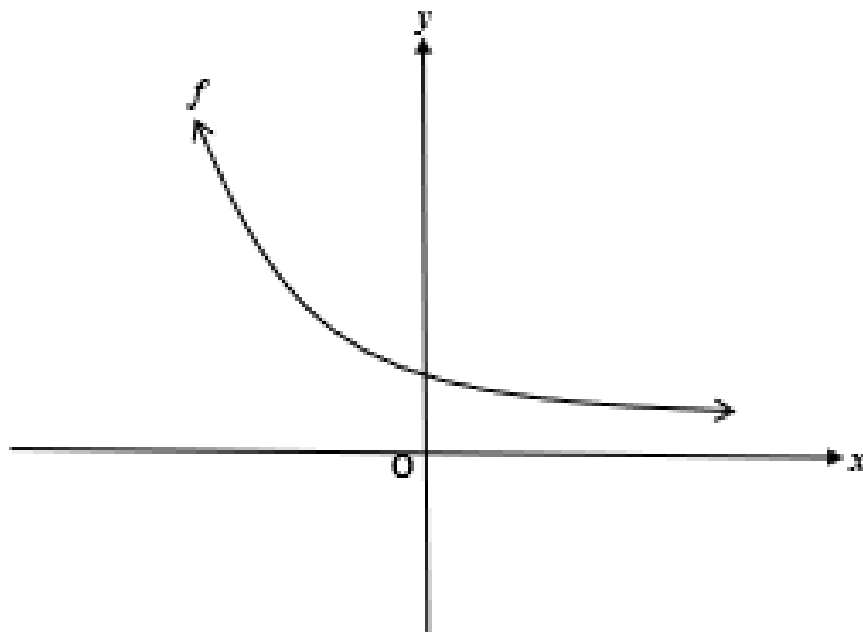

6.1.1	Determine the equation of h	(2)
6.1.2	Determine the coordinates of point A.	(2)
6.1.3	Determine the equation of f.	(3)
6.1.4	Determine the equation of the asymptotes of $f(x+1)$	(2)

6.1.5 Write down the coordinates of the image of $D\left(-\dfrac{1}{2};0\right)$ if D is reflected about

the axis of symmetry $y = x + 3$ (2)

6.2 Draw a rough sketch of the graph $y = a - \dfrac{1}{b-x}$, $(a > 0, b < 0)$ (3)

[14]

QUESTION 7

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $\left(2; \dfrac{1}{9}\right)$ lies on f.

7.1	Determine the value of k	(2)
7.2	Write down the range of f.	(1)
7.3	Explain the transformation of f to f^{-1}.	(1)
7.4	Determine the equation of f^{-1} in the form $y = \ldots$	(2)
7.5	Sketch the graph of f^{-1}. Indicate on your graph the coordinates of ONE point.	(3)
7.6	Prove that $\left[f(x)\right]^2 - \left[f(-x)\right]^2 = f(2x) - f(-2x)$.	(3)

[12]

QUESTION 8

8.1 Given: $f(x) = x^2 - 3$

Use first principles to find $f'(x)$. (5)

8.2 Determine $\dfrac{dy}{dx}$ if:

$$y = \frac{9x^4 - 6}{3x}$$ (3)

8.3 Evaluate leave your answer in a surd form.

$$\frac{d}{dx}\left[\frac{\sqrt[3]{x^3} - 2x\sqrt{x}}{3x}\right]$$ (4)

8.4 Given $f(x) = x^2 - 2x + 1$ and the gradient of the tangent at the point of contact is 3 determine the x-coordinate(s) at the point of contact. (2)

[14]

QUESTION 9

9. A cubic function has the equation $f(x) = -x^3 + 5x^2 - 7x + 3$.

9.1.1 Determine the coordinates of y-intercept. (1)

9.1.2 Determine the x-intercepts. (4)

9.1.3 Determine the coordinates of the turning point. (4)

9.1.4 Sketch the graph of f. (4)

9.1.5 Determine the x-value of point of the inflection. (3)

9.1.6 Sketch the graph of $f''(x)$ on the same set of axes. (2)

9.1.7 Write down the value(s) of x for which the graph of f is concave up. (2)

9.1.8 For which value(s) of x will $f'(x).f(x) \geq 0$? (2)

9.2   The graph of $f(x) = x^3 + 3x^2 + 24x + 20$ is shown below. The straight line with equation $y = -8$ touches the graph of $f(x)$ at the turning point $Q(2;-8)$ and crosses the graph of $f(x)$ at point P, as shown in the figure below. Determine the coordinates of P.   (4)
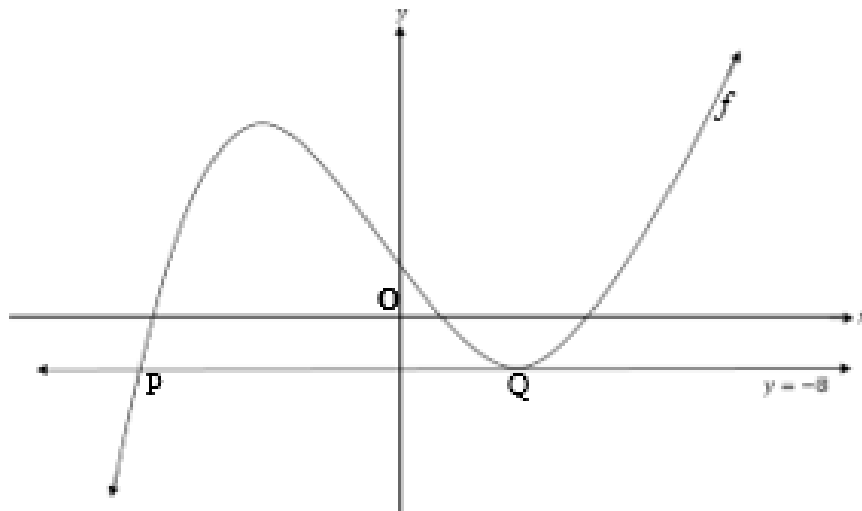
9.3   The graph of the derivative f' of a function f is shown.

$y = f'(x)$
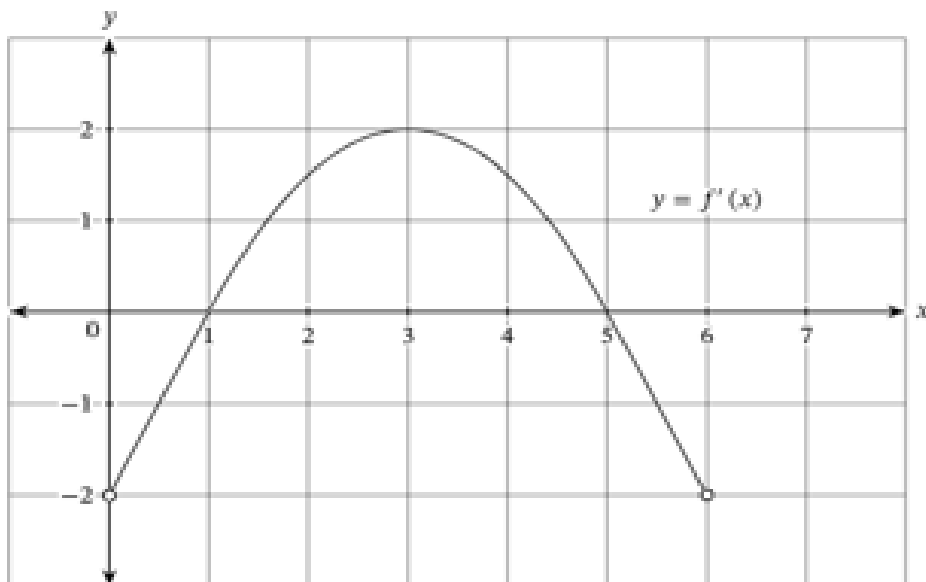
9.3.1   Determine the x values at the turning points of the graph f.   (2)

9.3.2   On what intervals is f decreasing?   (4)

[32]

TOTAL: 150