NOTES: Saturday 04/02/2023 Sequences and Series

1. Consider the quadratic number pattern: $-20\ ;\ -9\ ;\ 0\ ;\ 7\ ;...$

1.1 Determine the n^{th} term . (4)

1.2 Determine the position and the value of the term with the highest value. (3)

Solution:

1.1	$-20; -9; 0; 7; \ldots$ $11 \quad 9 \quad 7$ $-2 \quad -2$ $2a = -2 \qquad 3(-1) + b = 11$ $-1 + 14 + c = -20$ $a = -1 \qquad b = 14$ $c = -33$ $\therefore T_n = -n^2 + 14n - 33$	✓value of a ✓ value of b ✓ value of c ✓ T_n (4)
1.2	$n = \frac{-b}{2a}$ $= \frac{-14}{2(-1)}$ $n = 7$ $\therefore T_7 = -(7)^2 + 14(7) - 33$ $= 16$	✓ $\frac{-14}{2(-1)}$ ✓value of n ✓Value of T_7 (3)
		[7]

2. Given the following arithmetic sequence: $13 ; 8 ; 3 ; ...$

2.1 Determine the value of the 50th term. (3)

2.2 Calculate the sum of the first fifty terms. (2)

Solution:

2.1	$13;8;3;...$ $a=13$ and $d=-5$ $T_n=a+(n-1)d$ $T_{50}=13+(50-1)(-5)$ $T_{50}=-232$	✓$d=-5$ ✓substitution from the correct formula ✓Answer (3)
2.2	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{50}=\frac{50}{2}[2(13)+(50-1)(-5)]$ $S_{50}=-5475$	✓Substitution from the correct formula ✓Answer (2)

3. Prove that: $a+a+d+a+2d+...$ (to n terms) $=\frac{n}{2}[2a+(n-1)d]$ (4)

Solution:

3.	$S_n=a+(a+d)+(a+2d)+...+(l-2d)+(l-d)+l...(1)$ $S_n=l+(l-d)+(l-2d)+...(a+2d)+(a+d)+a...(2)$ $2S_n=(a+l)+(a+l)+(a+l)+...(a+l)+(a+l)+(a+l)$ $\therefore 2S_n=n(a+l)$ $\therefore S_n=\frac{n}{2}(a+l)$ $\therefore S_n=\frac{n}{2}[a+a+(n-1)d]$ $\therefore S_n=\frac{n}{2}[2a+(n-1)d]$	✓equation 1 and 2 ✓$2S_n=n(a+l)$ ✓dividing by 2 ✓substitution of l (4)

4. Consider the geometric series: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+...$

For which value(s) of m will the series converge? (3)

4.2 It is given that: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+...=\frac{27}{7}$

Calculate the value of m (3)

4.3 Determine the value of n if:

$$\sum_{r=1}^{n} 5.2^{1-r}=\frac{630}{64}$$ (6)

Marking Guideline

4.1	$3+m+\frac{m^2}{3}+\frac{m^3}{9}+...$ $r=\frac{m}{3}$ $-1<r<1$ $-1<\frac{m}{3}<1$ $-3<m<3$	✓$r=\frac{m}{3}$ ✓substitution of r ✓Answer (3)
4.2	$S_\infty=\frac{a}{1-r}$ $\frac{27}{7}=\frac{3}{1-\frac{m}{3}}$ $27-\frac{27m}{3}=21$ $27-9m=21$ $6=9m$ $\therefore m=\frac{6}{9}=\frac{2}{3}=0,67$	✓substitution ✓simplification ✓Answer (3)

4.3	$\sum_{r=1}^{n} 5.2^{1-r} = 5 + \frac{5}{2} + \frac{5}{4} + ...$ $S_n = \frac{a(1-r^n)}{1-r}$ $\frac{630}{64} = \frac{5\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$ $\frac{63}{64} = 1-\left(\frac{1}{2}\right)^n$ $\therefore \left(\frac{1}{2}\right)^n = \frac{1}{64}$ $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$ $n = 6$	✓ expansion to THREE terms ✓ $a = 2$ *and* $r = \frac{1}{2}$ ✓ subst into the correct formula ✓ simplification: $\frac{63}{64} = 1-\left(\frac{1}{2}\right)^n$ ✓ same bases: $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$ ✓ answer (6)
		[21]

MARCH 2015

QUESTION 3

Consider the infinite geometric series: 45 + 40,5 + 36,45 + …

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

QUESTION 3

3.1	$r=\frac{40,5}{45}=0,9$ $T_{12}=45(0,9)^{12-1}$ $=14,12147682...$ $=14,12$	✓$r=0,9$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (3)
3.2	$r=0,9$ $-1<0,9<1$	✓answer/*antwoord* (1)
3.3	$S_\infty=\frac{45}{1-0,9}$ $S_\infty=450$	✓ substitution/*substitusie* ✓ 450 (2)
3.4	$S_\infty-S_n<1$ $S_\infty-S_n=450-\frac{45\left(1-(0,9)^n\right)}{1-0,9}$ $S_\infty-S_n=450-450\left(1-(0,9)^n\right)$ $450(0,9)^n<1$ $(0,9)^n<\frac{1}{450}$ $\log(0,9)^n<\log\frac{1}{450}$ $n.\log(0,9)<\log\frac{1}{450}$ $n>\frac{\log\frac{1}{450}}{\log(0,9)}$ $n>57,98...$ Smallest value/*Kleinste waarde:* $n = 58$	✓$450-\frac{45\left(1-(0,9)^n\right)}{1-0,9}$ ✓$(0,9)^n=\frac{1}{450}$ ✓ introducing/*gebruik* logs ✓making n the subject/*maak n die onderwerp* ✓$n=58$ (5) **[11]**