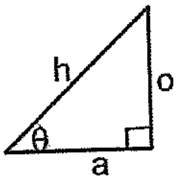
MODULE 7: Trigonometry

Revision

$\sin\theta = \frac{o}{h}$

$\cos\theta = \frac{a}{h}$

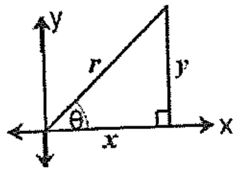
$\tan\theta = \frac{o}{a}$

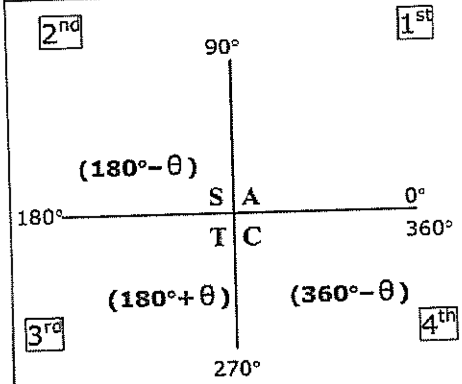


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$





Second quadrant

$\sin(180°-\theta)=\sin\theta$

$\cos(180°-\theta)=-\cos\theta$

$\tan(180°-\theta)=-\tan\theta$

Third quadrant

$\tan(180°+\theta)=\tan\theta$

$\cos(180°+\theta)=-\cos\theta$

$\sin(180°+\theta)=-\sin\theta$

Fourth quadrant

$\cos(360°-\theta)=\cos\theta$

$\sin(360°-\theta)=-\sin\theta$

$\tan(360°-\theta)=-\tan\theta$

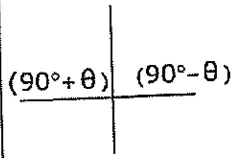
Co-functions

$\sin(90°-\theta)=\cos\theta$

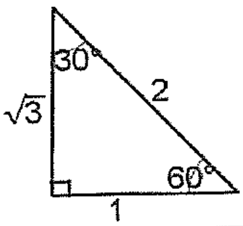
$\cos(90°-\theta)=\sin\theta$

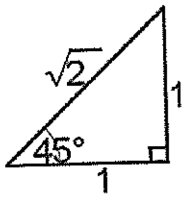
$\sin(90°+\theta)=\cos\theta$

$\cos(90°+\theta)=-\sin\theta$



Special Traingles **KNOW!**





Negative angles

$\cos(-A) = \cos A$

$\sin(-A) = -\sin A$

$\tan(-A) = -\tan A$

Study the following theory well.

Compound-angle identities

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

Double-angle identities

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$
$= 1 - 2\sin^2 A$
$= 2\cos^2 A - 1$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

Identities

$\sin^2 A + \cos^2 A = 1$

$\therefore \cos^2 A = 1 - \sin^2 A$

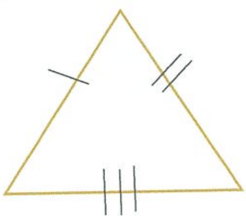
and $\sin^2 A = 1 - \cos^2 A$

$\tan A = \frac{\sin A}{\cos A}$

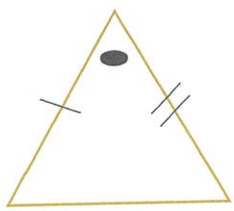
Negtive angle (Add 360°) 1. $\sin(-120°)$ $= \sin 240°$ $= \sin(180°+60°)$ $= -\sin 60°$ $= \frac{-\sqrt{3}}{2}$	**Angle greater than 360° (subtract 360°)** 2. $\tan 420°$ $= \tan 60°$ $= \frac{\sqrt{3}}{1}$
Not a reduction formulae (+ or – 360°) 1. $\sin(540° - \theta)$ $= \sin(180° - \theta)$ $= \sin\theta$ 2. $\tan(\theta - 360°)$ $= \tan\theta$ 3. $\cos(\theta - 180°)$ $= \cos(\theta + 180°)$ $= -\cos\theta$	**Exception** $(\theta - 90°)$ (take out a negative) 1. $\sin(\theta - 90°)$ $= \sin -(90° - \theta)$ $= -\sin(90° - \theta)$ $= -\cos\theta$
Square **(square goes outside the brackets)** 1. $\sin^2(180° - \theta)$ $= [\sin(180° - \theta)]^2$ $= [-\sin\theta]^2$ $= \sin^2\theta$	**Cofunctions are equal if their angles add up to 90°** 1. $\sin 30° = \cos 60°$ 2. $\cos 20° = \sin 70°$

USE COSINE WHEN YOU ARE GIVEN:

1. SSS

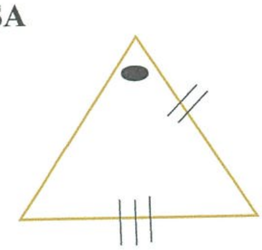


2. SAS (TWO SIDES AND AN INCLUDED ANGLE)

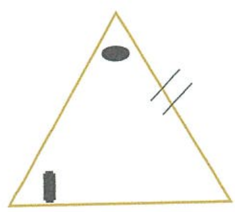


USE SINE WHEN YOU ARE GIVEN:

1. SSA



2. AAS



MODULE 8: Sine- Cosine- and Area Rule

1. **Area rule** (S∠S)

$$\text{Area } \triangle ABC = \frac{1}{2}ab\sin C$$

2. **Sine rule** (SS∠), (∠∠S)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

3. **Cosine rule** (SSS), (S∠S)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

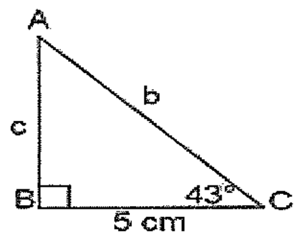
and

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Examples (Solving Δ's)

1. **In right-angled Δ:**
(use trig. ratios)

Determine b and c



1.1 $\frac{c}{5} = \tan 43°$

$\therefore c = 5\tan 43° = 4{,}7$ cm

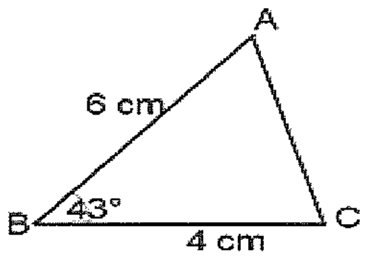
1.2 $\frac{5}{b} = \cos 43°$

$\therefore b = \frac{5}{\cos 43°} = 6{,}8$ cm

Examples

In **non** right-angled Δ's

2. **Area rule:** (S∠S)

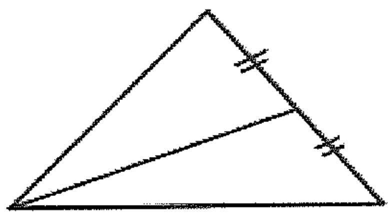


$\text{Area } \triangle ABC = \frac{1}{2}ac\sin B$

$\text{area.} = \frac{1}{2} \times 4 \times 6 \sin 43°$

$= 8{,}2 \text{ cm}^2$

median – divide area of Δ in two equal parts



TRIG NOTES 16/03/2024

EXAMPLE

5.3 Given the expression:

$$\frac{\sin(90°+x).\cos(-x).\tan^2(540°+x)}{\cos(180°-x).\sin(x-90°)-1}$$

5.3.1 Simplify the expression fully. (5)

5.3.2 For which value(s) of x, is the identity undefined? (3)

5.3.1	$=\frac{(\cos x)(\cos x)(\tan^2 x)}{(-\cos x)(-cox)-1}$ $=\frac{\cos^2 x.\tan^2 x}{\cos^2 x-1}$ $=\frac{\cos^2 x.\frac{\sin^2 x}{\cos^2 x}}{\cos^2 x-1}$ $=\frac{\sin^2 x}{-\sin^2 x}$ $=-1$	✓ $\cos x.\cos x$ ✓ $\tan^2 x$ ✓ $-\cos x.-\cos x$ ✓ $-\sin^2 x$ ✓ -1 (5)
5.3.2	$\cos^2 x-1=0$ $\cos x \quad =\pm1$ $ref\ angle=0°$ $\therefore x=0°$ or $x=180°$ or $x=360°$	✓ $x=0°$ ✓ $x=180°$ ✓ $x=360°$ (3)

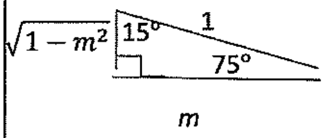
EXAMPLE

5.2 If $Cos75°=m$, express the following in terms of m, show all your work:

5.2.1 $\sin 15°$ (2)

5.2.2 $\tan 15°$ (2)

5.2.3 $\cos 105°$ (2)

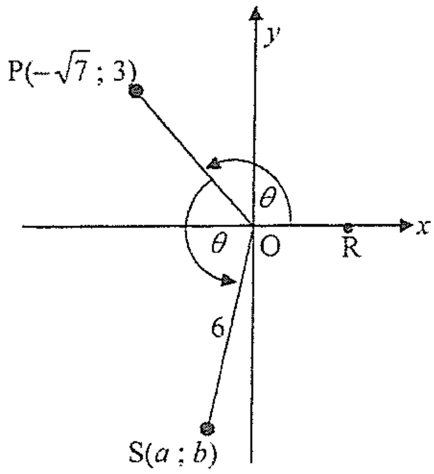
5.2.1	 $r^2 = x^2 + y^2$ *pyth* $y^2 = 1 - m^2$ $y = \sqrt{1-m^2}$ $\sin 15° = \cos 75° = m$	✓method ✓ m (2)
5.2.2	$\tan 15° = \dfrac{m}{\sqrt{1-m^2}}$	✓✓ $\dfrac{m}{\sqrt{1-m^2}}$ (2)
5.2.3	$[\cos 105°]$ $= [\cos(180° - 75°)]$ $= (-\cos 75°)$ $= -m$	✓ $-\cos 75°$ ✓ $-m$ (2)

DO THIS EXAMPLE

$$\frac{2\sin 510° - \cos 340° \cdot \cos(-20°)}{\cos^2 110°}$$

QUESTION 5

5.1 P$(-\sqrt{7}\,;3)$ and S$(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $\hat{POR} = \hat{POS} = \theta$ and OS = 6.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

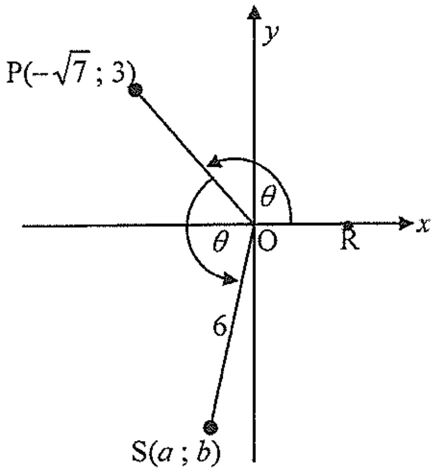
5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x - 1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15° - 1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

QUESTION/*VRAAG* 5



5.1.1	$\tan\theta = -\dfrac{3}{\sqrt{7}}$	✓answ/*antw* (1)
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\dfrac{3}{4}$	✓reduction/ *reduksie* ✓OP = 4 ✓answ/*antw* (3)
5.1.3	$\dfrac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2\theta)$ $= 6 - 12\left(\dfrac{3}{4}\right)^2$ $= \dfrac{24}{4} - \dfrac{27}{4}$ $= -\dfrac{3}{4}$ **OR/*OF*** $\dfrac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2\theta - 1)$ $= 12\left(\dfrac{-\sqrt{7}}{4}\right)^2 - 6$ $= \dfrac{21}{4} - \dfrac{24}{4}$ $= -\dfrac{3}{4}$ **OR/*OF***	✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\sin\theta = \dfrac{3}{4}$ ✓answ/*antw* (4) ✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \dfrac{-\sqrt{7}}{4}$ ✓answ/*antw* (4)

QUESTION 5

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

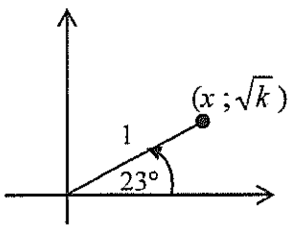
$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$ (6)

5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$	✓identity/*identiteit* ✓ $\cos^2 23°$ ito/*itv k* ✓ answ/*antw* (3)
	OR/*OF*	
	$x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \frac{\sqrt{1-k}}{1} = \sqrt{1-k}$ 	✓ $x^2 = 1 - k$ ✓ x ito/*itv k* ✓ answ/*antw* (3)
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\frac{\sin 23°}{\cos 23°}$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
	OR/*OF*	
	$\tan(-23°) = -\tan 23°$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.2	$\frac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \frac{-4\sin x.\cos x}{\sin 30°}$ $= \frac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

	OR/OF $\dfrac{4\cos x.(-\sin x)}{(\sin 30^\circ \cos x - \cos 30^\circ \sin x)\cos x + (\cos 30^\circ \cos x + \sin 30^\circ \sin x)\sin x}$ $= \dfrac{-4\sin x \cos x}{(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x)\cos x + (\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x)\sin x}$ $= \dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}\cos^2 x + \frac{1}{2}\sin^2 x}$ $= \dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}(\cos^2 x + \sin^2 x)}$ $= \dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}(1)}$ $= -8\cos x \sin x$ $= -4(2\sin x \cos x)$ $= -4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\frac{1}{2}\cos^2 x + \frac{1}{2}\sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form */ dubbel sin form* ✓ answ/*antw* (6)