NOTES SEQUENCES AND SERIES:

1. Quadratic number pattern: (Second difference is a constant)

- General Term: $T_n = an^2 + bn + c$
- There is NO formulae for the Sum of a Quadratic number Pattern:
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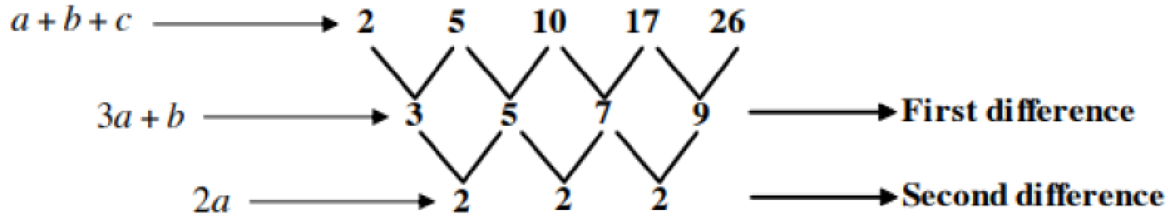
So, consider the previous number pattern: 2; 5; 10; 17; 26; ……..



It is clearly a quadratic number pattern because it has a constant second difference. You can now proceed as follows:

$2a = 2$ $\quad$ $3a + b = 3$ $\quad$ $a + b + c = 2$

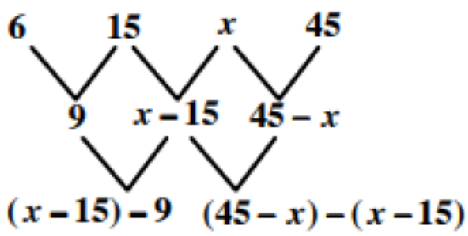
$\therefore a = 1$ $\quad$ $\therefore 3(1) + b = 3$ $\quad$ $\therefore 1 + 0 + c = 2$

$\therefore b = 0$ $\quad$ $\therefore c = 1$

Therefore the general term is $T_n = n^2 + 0n + 1 = n^2 + 1$

EXAMPLE

$6 ; 15 ; x ; 45 ; ....$ is a quadratic number pattern (sequence). Determine the value of x.



$(x-15) - 9 = (45 - x) - (x - 15)$

$\therefore x - 24 = 45 - x - x - 15$

$\therefore x - 24 = 60 - 2x$

$\therefore 3x = 84$

$\therefore x = 28$

EXAMPLE

The constant second difference of the quadratic number pattern:
$4 ; x ; 8 ; y ; 20 ; ....$ is 2.

(a) Determine the value of x and y.

(b) Determine which term equals 125.

DO THE SOLUTION

SEQUENCES AND SERIES (QUADRATIC NUMBER PATTERN

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; …

3.1.1 Show that the general term of this number pattern is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\ 527$? (4)

2.2 The following information is given about a quadratic number pattern:

$T_1 = 3$, $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)

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QUESTION 3

A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

3.1 Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$. (6)

3.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)

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SEQUENCES AND SERIES (QUADRATIC NUMBER PATTERN

QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

3.2 Determine the value of the 60^{th} term of this number pattern. (2)

3.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. (3)

3.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

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QUESTION 3/*VRAAG* 3

3.1.1	3 ; 7 ; 12 ; 18 First diff: 4 ; 5 ; 6 Second diff: 1 ; 1 $2a=1$ $a=\frac{1}{2}$ $3a+b=4$ $3\left(\frac{1}{2}\right)+b=4$ $b=\frac{5}{2}$ $a+b+c=3$ $\frac{1}{2}+\frac{5}{2}+c=3$ $c=0$ $T_n=\frac{1}{2}n^2+\frac{5}{2}n$	✓ $2a=1$ ✓ $3\left(\frac{1}{2}\right)+b=4$ ✓ $\frac{1}{2}+\frac{5}{2}+c=3$ (3)
3.1.2	$13\ 527=\frac{1}{2}n^2+\frac{5}{2}n$ $n^2+5n-27\ 054=0$ $(n-162)(n+167)=0$ $n=162$ or $n=-167$ $T_{161}=13\ 363$ $\therefore T_{161}+164=13\ 527$ 164 must be added. **OR/*OF*** $T_n=3+$ sum of 1st differences $13527=3+4+5+...+n$ $\frac{n-3+1}{2}[3+n]=13527$ $n^2+n-27060=0$ $(n+165)(n-167)=0$ $n=164$	✓ $13\ 527=\frac{1}{2}n^2+\frac{5}{2}n$ ✓ standard form ✓ answers for n ✓ 164 (4) **OR/OF** ✓ $13527=3+4+5+...+n$ ✓ $n^2+n-27060=0$ ✓ answers for n ✓ 164 (4)

SEQUENCES AND SERIES (QUADRATIC NUMBER PATTERN

2.2.1	$T_1 = 3\,;\, T_2 - T_1 = 9$ and $T_3 - T_2 = 21$
	
	3 12 33 66 111
	  9 21 33 45
	   12 12 12
	
	$\therefore T_5 = 3+9+21+33+45 = 111$
	
	OR/*OF*
	$2a = 12$
	$a = 6$
	$3(6)+b = 9$
	$b = -9$
	$6-9+c = 3$
	$T_5 = 6(5)^2 - 9(5) + 6 = 111$
		✓ constant second diff = 12
		✓ first differences : 33 and 45
		(2)
		
		OR/*OF*
		✓ constant second diff = 12
		
		✓substitute 5 (2)

2.2.2	$2a = 12$ $a = 6$ $3(6)+b = 9$ or $5\times 6 + b = 21$ $b = -9$ $6-9+c = 3$ $c = 6$ $T_n = 6n^2 - 9n + 6$	✓ $2a = 12$ ✓ $3(6)+b = 9 / 5\times 6 + b = 21$ ✓ $6-9+c = 3$ (3)
2.2.3	$T_n' = 12n - 9 > 0$ $n > \frac{3}{4}$ $\therefore$ T_n is increasing for $n \in N$ **OR/*OF*** $n = -\frac{b}{2a} = -\frac{-9}{2(6)}$ $n = \frac{3}{4}$ $\therefore$ min at $n = 1$ for $n \in N$ $\therefore$ T_n is increasing for $n \in N$	✓ $T_n' = 12n - 9$ ✓ $n > \frac{3}{4}$ ✓ increasing for $n \in N$ (3) **OR/*OF*** ✓ $n = -\frac{b}{2a} = \frac{9}{2(6)}$ ✓ $n = \frac{3}{4}$ ✓ increasing for $n \in N$ (3)
		[16]

SEQUENCES AND SERIES (QUADRATIC NUMBER PATTERN

<u>**MAY / JUNE 2023 / MEI/JUNIE 2023**</u>

QUESTION 3/*VRAAG* 3

3.1	$x \; ; \; x \; ; T_3 \; ; \ldots$	
	first differences: $0 \qquad T_3 - x$	
	second difference: 10	
	$2a = 10 \qquad 3a + b = 0$	✓ $2a = 10$
	$a = 5 \qquad b = -15$	✓ $3a + b = 0$
	$T_3 - x - 0 = 10$	
	$\therefore T_3 = x + 10$	✓ $T_3 = x + 10$
	$2x + T_3 = 28$	✓ $2x + T_3 = 28$
	$2x + x + 10 = 28$	
	$3x = 18$	
	$x = 6$	✓ $x = 6$
	$a + b + c = 6$	
	$5 - 15 + c = 6$	✓ $5 - 15 + c = 6$
	$c = 16$	(6)
	$\therefore T_n = 5n^2 - 15n + 16$	
	OR/*OF*	**OR/*OF***
	$2a = 10$	✓ $2a = 10$
	$\therefore a = 5$	
	$T_1 = a + b + c \qquad T_2 = 4a + 2b + c \qquad T_3 = 9a + 3b + c$	
	$= 5 + b + c \qquad = 20 + 2b + c \qquad = 45 + 3b + c$	
	$5 + b + c = 20 + 2b + c$	✓ $5 + b + c = 20 + 2b + c$
	$b = -15$	
	$T_1 = -10 + c \qquad T_2 = -10 + c \qquad T_3 = c$	✓ $T_1 = -10 + c$
		✓ $T_2 = -10 + c$
	$T_1 + T_2 + T_3 = -10 + c - 10 + c + c$	
	$28 = 3c - 20$	✓ $28 = 3c - 20$
	$c = 16$	✓ $c = 16$ (6)

SEQUENCES AND SERIES (QUADRATIC NUMBER PATTERN

3.2	$T_n = 5n^2 - 15n + 16$ $216 = 5n^2 - 15n + 16$ $5n^2 - 15n - 200 = 0$ $n^2 - 3n - 40 = 0$ $(n-8)(n+5) = 0$ $n = 8$ or $n \neq -5$ $\therefore T_8 = 216$	✓ equating ✓ standard form ✓ $n = 8$ (3)
		[9]

QUESTION 3/*VRAAG* 3

3.1	$3a + b = 7$ $3 + b = 7$ $b = 4$ **OR/OF** $T_2 - T_1 = 7$ $4 + 2b + 9 - (1 + b + 9) = 7$ $b = 4$	✓ $3a + b = 7$ ✓ $3 + b = 7$ (2) **OR/OF** ✓ $T_2 - T_1 = 7$ ✓ substitution (2)
3.2	$T_n = n^2 + 4n + 9$ $T_{60} = (60)^2 + 4(60) + 9$ $= 3849$ Answer only: full marks	✓ substitution ✓ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p = 2p + 5$ Answer only: full marks **OR/*OF*** First difference: 7 ; 9 ; 11 ; … $T_n = a + (n-1)d$ $T_p = 7 + (p-1)(2)$ $T_p = 2p + 5$	✓first difference ✓ 2 ✓ $2p + 5$ (3) **OR/*OF*** ✓first difference ✓ 2 ✓ $2p + 5$ (3)
3.4	$157 = 2p + 5$ $p = 76$ $\therefore$ Between T_{76} and T_{77} **OR/OF** $T_{n+1} - T_n = 157$ $(n+1)^2 + 4(n+1) + 9 - (n^2 + 4n + 9) = 157$ $n^2 + 2n + 1 + 4n + 4 + 9 - n^2 - 4n - 9 = 157$ $2n = 152$ $n = 76$ $\therefore$ Between T_{76} and T_{77}	✓ $157 = 2p + 5$ ✓ $p = 76$ ✓ T_{76} and T_{77} (3) **OR/OF** ✓ $T_{n+1} - T_n = 157$ ✓ $n = 76$ ✓ T_{76} and T_{77} (3)
		[10]